

# **Analysis of the Elastic Recovery in the Forming of Thick Plates for Shipbuilding Structures**

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**Abstract.** In a considerable number of industries, structures are fabricated from thick plates such as in shipbuilding, e.g., a ship's hull is built up from a series of transverse and longitudinal frames (0,5 to 1 in. thick) to form the outer skin of the hull. In general, the radii of curvature are large (greater than 15 ft. radius), as a result considerable difficulty is experienced in forming double curvature plates. One of the main problems for this is the elastic recovery (spring-back) of the plate after bending process has taken place. This causes trouble when fitting the individual sections of a ship, and consequently rectification methods are required. Needless to say, such methods are time-consuming and costly , as well as introducing residual stresses into the structure.

Therefore, the main aim of this paper is to present a theoretical method whereby the elastic recovery (spring-back) factor in bending the plate can be predicted from the basic properties of the sheet metal and tool geometry. The influence of the work hardening and the cold working characteristics have also

been taken into account. Previous experiments were produced here for comparison purposes and were found to be in reasonable agreement, making it more reliable for application in the bending process without many correction factors. Other parameters associated with the process are also examined.

## 1. Introduction

Several industries employ thick plates for the manufacture of their goods, such as shipbuilding and boiler making are the most obvious examples. A large number of these plates have to be curved in two directions; in particular those in the bow and stern sections of the ship. Normally, the radii of curvature are large, but the magnitude of the curvature may vary gradually over the area of the plate, and this leads to many difficulties in forming the plate accurately to its desired shape. As a result, a number of rectification processes may be needed, such as cutting *in situ* at the berth, the deposition of excess weld material and the use of force to deform the plates on assembly. These methods normally, are undesirable because they are time consuming and costly, and add considerably to the assembly time of the hull, as well as introducing residual stresses [1,2].

Generally, the bending of single curvature plates is simple and can be carried out on conventional three-roll bending machines. These machines can also be used to form double curvature plates. This method is rather imprecise and only achieved by experienced operators capable of altering the angle of feed of the plates into the rolls and/or by judiciously inserting pieces of wood, etc, between the rolls to produce the required double curvature. The bent plates are checked by means of templates made up from strips of wood and the forming is done largely on a trial and error basis, relying heavily on the skill of the operators [3]. In addition, double curvature forming is extremely time-consuming. Clearly, the roll bending process does not lend itself to modifications that would enable double curvature plates to be formed more accurately and automatically.

There is, therefore, a need for more accurate and versatile methods for bending of plates with double curvature, for application in ship building and similar industries [2,4,5,6]. Needless to say, the development of numerical control machines for the associated processes of flame cutting and frame bending,

highlights the need for such a plate double curvature bending machine. The increase in accuracy achievable with such equipment may be fully exploited at the assembly stage of the ship's hull, etc. [7,8]. However, for cost reduction of the process, some suggestions have been made for presses in which the die profile is altered by rams or jacks to a suitable shape for each plate, but these clearly require some considerable development [2,9].

Further techniques which have received some attention are those of heat line bending [2,4,5] and squeeze bending, but these have received little attention [2,9,10].

It is a well known fact, that metal during the bending process suffers some elastic recovery after the unloading stage. The amount of spring-back depends on many factors, such as, material properties, tool geometry and operational conditions. Therefore, the purpose of this work is to establish the importance of these parameters in the single bending of thick plates.

## 2. Theoretical Consideration

In general, metal bending method is accomplished by many techniques which produce bending moment caused by the application of the bending load. Since all plate materials have a finite modulus of elasticity, then plastic deformation is followed by elastic recovery when unloading takes place. This phenomena is known as the spring-back factor ( $k_p$ ) as shown in Figure 1. In industrial applications this factor is measured by several means such as the variation in the bend angle before and after bending. Normally, it is expressed as the ratio between the final radius ( $R$ ) of bend and the final radius of the piece after bending ( $R_f$ ), or  $k_p = R/R_f$ . However, to develop a theory that predicts the spring-back factor for thick plates that takes into account the elastic and plastic characteristics of the material plates, and tool geometry, simplifying assumptions are made. The neutral axis is midway through the plate metal, also there is no stress existing at this point, the plane section remains plane after bending, the arc of bend is circular and the true stress-strain behavior can be represented by [11].

$$\sigma = C(B + \bar{\epsilon})^n \quad (1)$$

where  $C$  is the strength coefficient,  $n$ , work hardening exponent and  $B$  the cold working or residual strain.

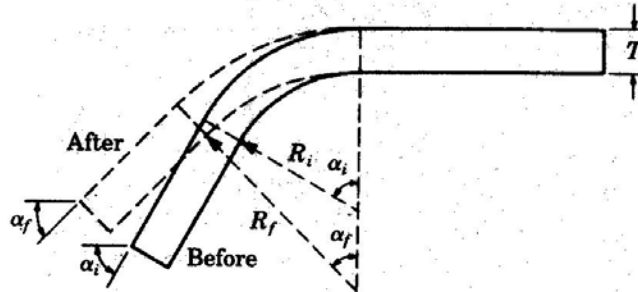


Figure 1 – Schematic illustration of spring-back in bending process.

A rectangular material plate section (width  $b$  and depth  $h$ ) subjected to an elastic-plastic moment ( $M_x$ ), where the neutral axis coincides with the centroidal axis, is given by [12,13]

$$M_x = 2 \int_0^{h_e/2} bdy \sigma_e y + 2 \int_{h_e/2}^{h/2} bdy \sigma_p y \quad (2)$$

where  $\sigma_e$  is the stress in the elastic region,  $\sigma_p$  is the stress in the plastic region,  $h_e$  is called elastic-plastic boundary ( $M_{yp} \leq M_x \leq M_p$ ), where  $M_{yp}$  is the moment at the instant of yielding at the extreme fiber, i.e., when  $h_e = h$ , and  $M_p$  is the plastic-moment capacity of the section.

The relation of the elastic stress ( $\sigma_e = E.e$ ) and strain ( $e = y/R$ ), and by substituting of these equations together with Eq.(1) into Eq.(2) and integrating the first term gives

$$M_x = \frac{bEhe^3}{12R} + 2bC \int_{h_e/2}^{h/2} \left(B + \frac{y}{R}\right)^n y dy \quad (3)$$

Now, the second term can be integrated by parts to yield the final expression for the elastic-plastic moment, and is given by

$$M_w = \frac{bEhe^3}{12R} + \frac{2bCR^2}{n+2} \left[ \left( B + \frac{h}{2R} \right)^{n+2} - \left( B + \frac{h_e}{2R} \right)^{n+2} \right] - \frac{2bCBR^2}{n+1} \left[ \left( B + \frac{h}{2R} \right)^{n+1} - \left( B + \frac{h_e}{2R} \right)^{n+1} \right] \quad (4)$$

Upon removing the load, spring-back will take place, and in order to restore the situation under load, an elastic moment has to be applied and is given by the following relation

$$M_e = \frac{Ebh^3}{12R_E} \quad (5)$$

Clearly, during the bending process the elastic-plastic moment according to Eq.(4) is in equilibrium with the formulated expression given by Eq.(5). By rearranging the terms, therefore, the final theoretical expression for the spring-back is given by

$$\frac{R}{R_f} = 1 - 8 \left( \frac{\sigma_e R}{Eh} \right)^3 - \frac{24C}{(n+2)E} \left( \frac{h}{R} \right)^{n-1} \left[ \left( \frac{BR}{h} + \frac{1}{2} \right)^{n+2} - \left( \frac{BR}{h} + \frac{\sigma_e R}{Eh} \right)^{n+2} \right] + \frac{24CB}{(n+1)E} \left( \frac{h}{R} \right)^{n-2} \left[ \left( \frac{BR}{h} + \frac{1}{2} \right)^{n+1} - \left( \frac{BR}{h} + \frac{\sigma_e R}{Eh} \right)^{n+1} \right] \quad (6)$$

The above expression will be termed here as the ‘‘General Formula’’. It is evident that the spring-back factor ( $R/R_f$ ) depends on the characteristics of the plate materials, particularly the work-hardening index ( $n$ ) and the cold working strain ( $B$ ), besides the elastic properties of the plate. This being in addition to the tool geometry.

In many plate materials the value of the residual strain ( $B$ ) in Eq.(1) is small and may be neglected, then a fair approximation of the final expression given in Eq.(6) for  $B = 0$  yields the following equation.

$$\frac{R}{R_f} = 1 - \frac{R}{R_E} = 1 - 8 \left( \frac{\sigma_0 R}{Eh} \right)^3 - \frac{3C}{(n+2)E} \left( \frac{h}{2R} \right)^{n-1} + \frac{24C}{(n+2)E} \left( \frac{h}{R} \right)^{n-1} \left( \frac{\sigma_0 R}{Eh} \right)^{n+2} \quad (7)$$

On the other hand, a situation when there is no hardening of the plate (elastic-perfectly plastic), then  $n = 0$ ,  $B = 0$  and  $C = \sigma_0$  (yield stress). Hence Eq.(7) can be reduced to the following expression for the spring-back

$$\frac{R}{R_f} = 1 - \frac{R}{R_E} = 1 - 3 \left( \frac{\sigma_0 R}{Eh} \right) + 4 \left( \frac{\sigma_0 R}{Eh} \right)^3 \quad (8)$$

This is the simplified version of the general formula, Eq.(6), which is in fact similar to that given in references [12-15].

Evidently from the previous equations, a spring-back factor  $(R/R_f) = 1$  indicates no spring-back and  $(R/R_f) = 0$  indicates complete elastic recovery, e.g., the use of springs in many industrial applications such as, automobile, trains, etc.

Table 1 – Mechanical properties of various sheet metals  $\bar{\sigma} = A \bar{\epsilon}^n$ .

Material	Properties Tensile strength ( $\sigma_y$ ) kg/mm <sup>2</sup> (MN/m <sup>2</sup> )	$E$ kg/mm <sup>2</sup> (GN/m <sup>2</sup> )	Elongation on 2 in. gauge length (%)	$A$ kg/mm <sup>2</sup> (MN/m <sup>2</sup> )	$n$
Aluminium	9.77 (95.8)	7340 (72.3)	40	20.95 (205)	0.27
Half-hard copper	22.68 (222)	12758 (125)	19	53.55 (525)	0.12
70/30 Brass	29.30 (287)	10545 (103)	62	86.63 (850)	0.57
Mild steel	40.00 (392)	20380 (200)	23	64.58 (633)	0.21
Hard steel	57.96 (568)	21090 (207)	4	—	—
Hard phosphor bronze	63.00 (618)	11246 (110)	5	—	—

### 3. Discussion of Results

Before any discussion is made, it is worth mentioning that in Figs. 2, 3, 4 and 5, the y-axis represents the spring-back factor  $(R/R_f)$  and the x-axis

represents the stiffness of the plate  $(\sigma \cdot R)/(E \cdot h)$ . Both of these axes are dimensionless, so that any plate material characteristics, thickness and tool geometry can be represented easily to estimate the spring-back factor. Needless to say, when bending a plate, the beam stiffness  $(\sigma \cdot R)/(E \cdot h)$  should be replaced by the plate stiffness  $[(1 - \mu^2) \cdot (\sigma \cdot R)/(E \cdot h)]$  where  $\mu$  is Poisson's ratio.

To facilitate comparison between the general formula of the spring-back, Eq.(6), and its simplifications, Eqs.(7) and (8), then they are plotted together and are shown in Fig. 2. It appears from the results predicted by the general formula that Eq.(6) falls between the curves of  $B=0$  and  $n=0$ . However, the curve of  $B=0$  and  $n \neq 0$  gives very high spring values for any beam stiffness  $(\sigma \cdot R/E \cdot h)$ , whereas the curve of the general formula, Eq.(6) resides closely to the curve of  $n=0$  and  $B=0$ , Eq.(8). Therefore, it can be concluded that the simplified expression given in Eq.(8) estimates the same value of the spring-back, particularly for the lower values of  $\sigma \cdot R/E \cdot h < 0,2$ . On the other hand, the expression of the spring-back given in Eq.(7) gives different results. This is due to the fact that the plate material only work hardened without any residual strains during the bending process, Figure 2. Therefore, care must be taken in the representation of the tensile test results, otherwise misleading prediction may be obtained.

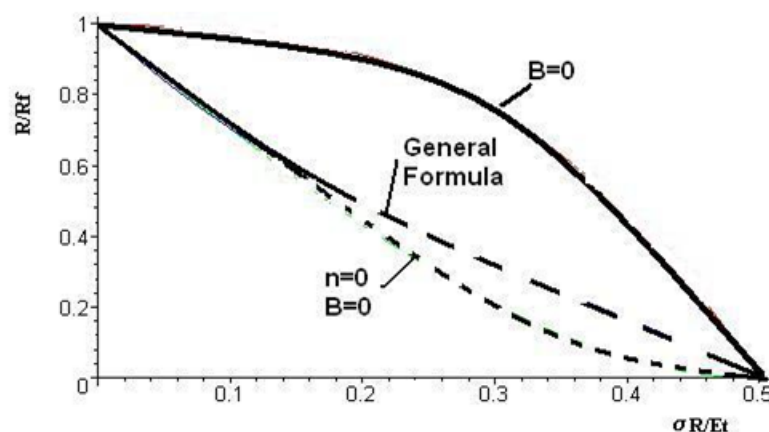


Figure 2 – Comparison between the General Formula and its simplifications.

It can be observed from Figure 3 that the behaviour of the material becomes more work hardened as the values of  $n$  becomes larger; thus producing larger spring-back values.

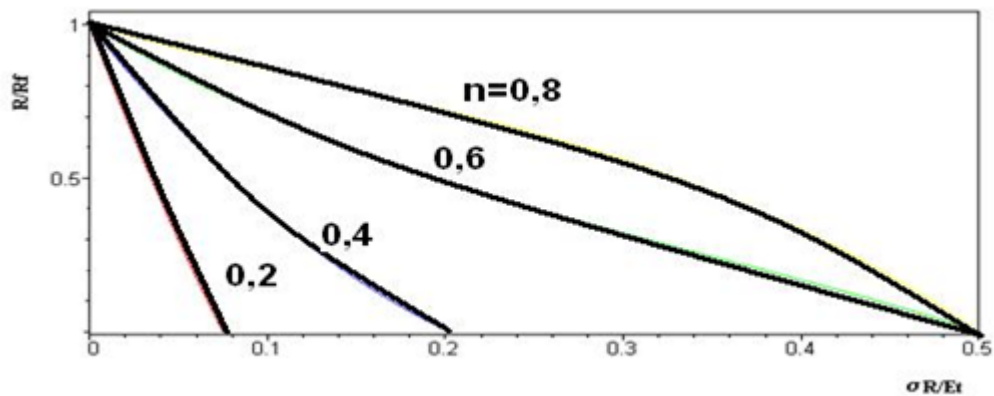


Figure 3 – Variation of the spring-back factor with the work hardening exponent of the plate material

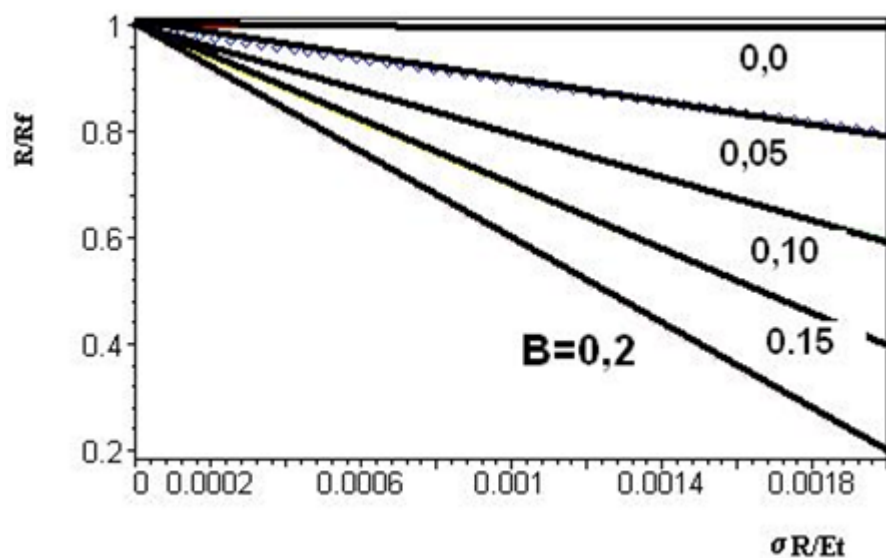


Figure 4 - Variation of the spring-back factor with cold working strain

On the other hand, the larger the values of cold working strain ( $B$ ) the smaller the spring-back, Figure 4. In other words, the material becomes more springy.

In this work, it was essential to reproduce some of the previously published work which was carried out on a variety of sheet metals having different work-hardening characteristics for comparison purposes. A list of them, together with mechanical properties along the rolling direction, is presented in Table 1 [12].

For the sake of interpretation, all the previous experimental results are plotted together with the theoretical curve for  $B = 0$  and  $n = 0$ , Eq. (8), shown in Figure 5. Close examination of this figure demonstrate that the experimental data of the spring-back factor for soft material resides below the theoretical curve. This may be due to the fact that the actual stress at a particular strain is greater than the assumed one. More springy materials exhibited greater experimental values of spring-back than the predicted theoretical results. This is caused by the fact that the elastic behaviour of the exotic materials is not perfect.

Other authors have suggested probable band values of spring-back which could be included in the figures. This band is mainly designed to accommodate the deviation of the experimental results [12-15]. Such graphical representations will certainly reduce the time for estimating the spring-back, thus reducing the cost and time for the tool development during the try out scheme.

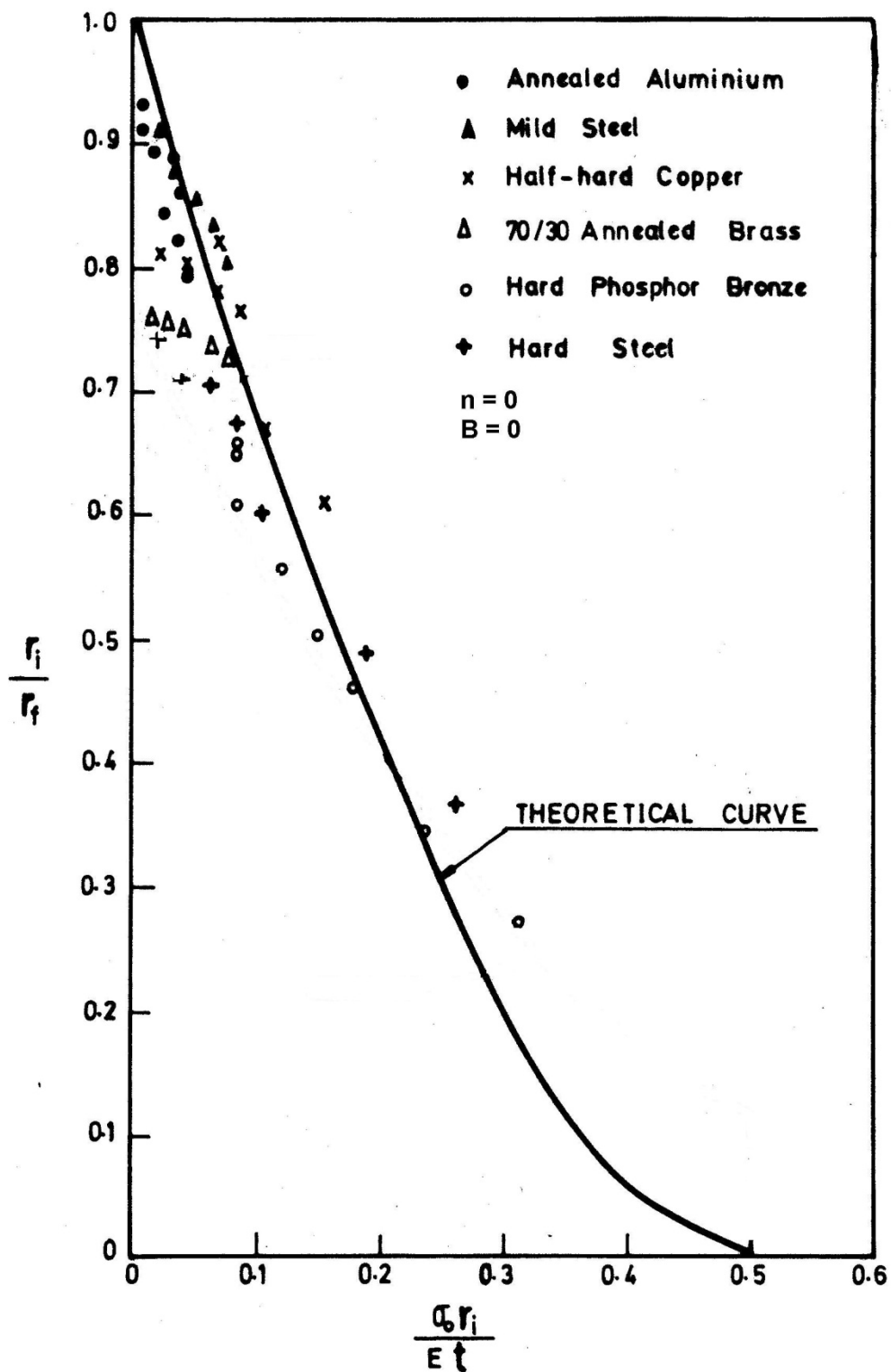


Figure 5 – Comparison between theoretical and experimental results for different materials.

#### **4. Conclusions**

In many plate/sheet bending operations, the well known phenomena of spring-back has always created problems for shipbuilding and other industries, and much tool development is required particularly for double curvature bending of plates.

The results presented here, show that it is possible to predict theoretically the spring-back factor with good accuracy. The developed general formula for predicting this factor appears to be more useful from the practical point of view. In addition, the simplified version of this formula may also be used to estimate approximately the quantity of the elastic recovery after bending for any given plate material and tool geometry. In addition, as mentioned previously the tensile test experiments must be represented as accurately as possible to lead to a better predication of the spring-back factor, consequently, reducing dramatically the tool development stage.

Finally, the present theoretical analysis could be considered low cost tooling development, and also for general industrial applications. Needless to say, it is also possible to develop from it fairly simple set of tables and formulae which can conveniently be used by press and tool designers.

#### **5. Acknowledgements**

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