State of the Art of Life Prediction Methods for Fatigue Design of Marine Structures

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ABSTRACT

In complex structural details, stress concentrations are emphasized by a tortuous stress-flow along the structural discontinuities, and both stress multiaxiality and elastic-plastic strain field are increased. Conventional methods are hardly valid without adjustments when very-high stress concentrations exist and the material local response is a repeated plastic deformation. The strain-based approach, which considers the elastic-plastic strain range as the governing load parameter, becomes the reference design method when fatigue failure occurs after relative low number of load cycles (low-cycle fatigue) – as stated in Radaj *et al.* (2006) and in Wang *et al*. (2006). On the other hand, stress multiaxiality may be dealt by referring to different criteria, generally oriented to the implementation of the traditional cumulative fatigue damage procedure based on *S-N* curves – as illustrated in the technical report of the ISSC Committee III.2 “Fatigue and Fracture” (2009).

The work here presented deals with a case study performed by the University of Trieste and Fincantieri for the improvement of a structural detail where weld joints are subject to proportional biaxial variable-amplitude loading leading to high plastic strains. After discussing how the *S-N* method should be modified to allow for stress multiaxiality and elastic-plastic strain, a critical examination is carried out following alternatives for effective equivalent stress hypothesis.

METHODS FOR THE FATIGUE ANALYSIS

Within the cumulative damage procedure, the fatigue design of any structural detail is performed by making use of a reference parameter s derived by the stress-strain field measured or calculated in the vicinity of the crack site. The fatigue check is performed by comparing s with the strength value S, which is endurable at the same number of cycles N. Fatigue capability is given in form of S-N curve. Depending on the chosen reference parameter, fatigue assessment is carried out by different methods.

The "nominal stress approach" is based on the concept of nominal stress σ_n, that is the far-field stress due to the forces and moments at the potential site of cracking. The nominal stress approach ($s = \sigma_n$) is a robust and straightforward "global approach" for assessing fatigue strength of any classified (i.e., typical) structural detail. Extensive experience has been acquired over the years in the use of such a very practical approach, to such an extent that it continues to be the primal method for fatigue assessments. The method has yet certain intrinsic limitations to be ascribed to a poor tracing of the decisive fatigue-strength influencing parameters. On the other hand, its formal simplicity makes it the right method for design codes and guidelines.

The nominal stress approach is replaced by a "local approach" when a high demand of accuracy is needed, as well as in the case of complex structures when the concept of nominal stress loses meaning or, at least, when an unambiguous nominal stress level is difficult to identify. In the local approaches, the fatigue strength of any structural detail is assessed by making reference to the intensity of the stress or strain field measured or calculated just in the area at risk of crack initiation − such a critical location is usually denoted a "hot spot". In general, analyses of structural discontinuities and details are not possible using analytical methods. Thus, the finite element analysis is mostly applied according to the local approach used for the fatigue assessment. Within the FE analysis, uncertainties in the computed stresses and predicted fatigue lives are mainly due to the element properties and sizes. Due to the sensitiveness to FE properties of the stress-strain field at the hot spot, different methods have been conceived for getting the more appropriate FE-calculated fatigue effective parameters.

The structural stress approach

The "structural stress approach" takes into account only that part of the local stress concentration which is related to the structural geometry. In plate-type structures, the structural stress σ_s is commonly defined as the sum of a membrane and a bending stress at the hot spot, excluding the local nonlinear stress peak at the notch. The decisive issue in any structural-stress based approach ($s = \sigma_s$) is how to obtain a significant structural stress value, that is a stress neglecting the notch effect but, at same time, effective in describing the macrostructural behaviour of the structural component, and so, reactive to very slight changes in the local configuration of the structural component.

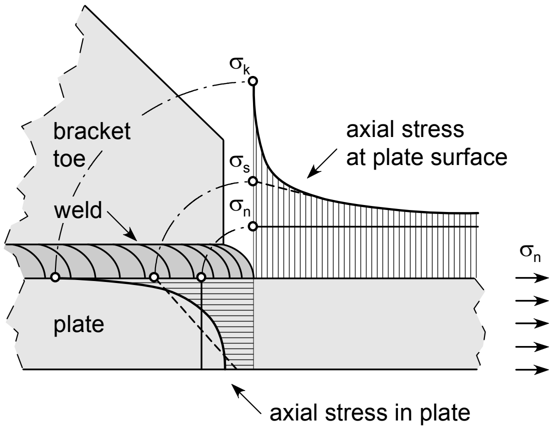

Figure 1. Stress types at a typical hot spot in a plate-type structure.

The definition of a procedure as unambiguously as possible for separating local configuration effect from the notch effect is the subject of today's investigations and the following procedures have been developed to derive the reference structural stress at the hot spot (Radaj *et al.*, 2006):

- the extrapolation of stresses to obtain a hot-spot stress;
- the equilibrium searching in a small volume at the hot spot to obtain an equilibrated structural stress, as in the Dong's method;
- the evaluation of an effective structural stress at a read out point properly located in the vicinity of the crack initiation point. In the method proposed by Xiao and Yamada (2004) the stress is measured 1 mm below the surface on the expected crack path; in the method proposed by Haibach, the stress is determined on the surface 2 mm apart from the weld toe.

In Fig. 1 the structural stress extrapolation at the weld toe of a bracket on a plate surface is shown (σ_k is the notch stress, σ_s the structural stress and σ_n the nominal stress). Figure shows that the structural stress may be derived as "hot-spot stress" σ_{hs} by two ways:

- from the stresses at plate surface (surface extrapolation);
- from the stresses in the cross section at the weld toe (through-thickness extrapolation).

The surface stress extrapolation is carried out on stresses read out on points located relatively far from weld to not feel weld-notch effect, but sufficiently close to the potential crack site to completely catch the influence of the "geometry effect". There are a lot of proposals for the location of those points and in most cases the plate thickness is a suitable parameter to position the evaluation points. In the most common standards, the hot-spot stress is defined as the stress obtained from FE analysis by linear extrapolation on the read out points located at 0.5 t and 1.5 t from the weld toe (being t the plate thickness). The ratio between the hot-spot stress and the nominal stress is defined as the "hot-spot stress concentration factor" K_{hs}.

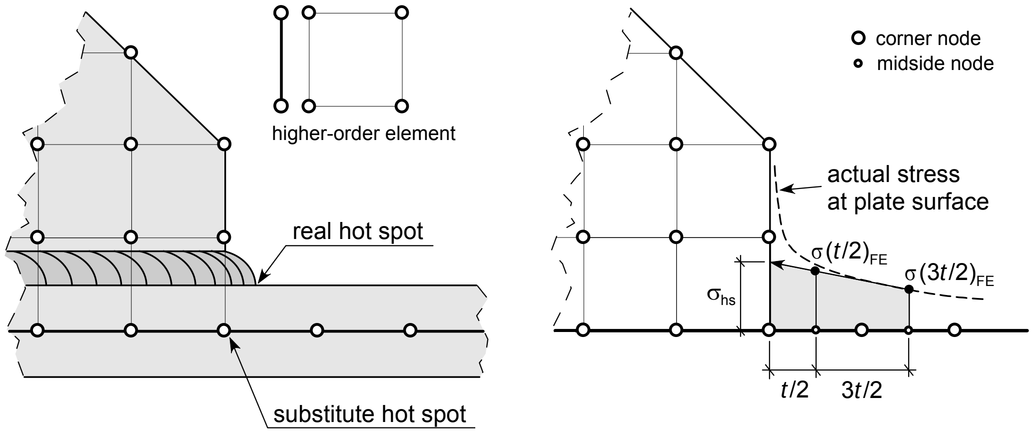

Figure 2. FE model for a typical hot spot and evaluation of the hot-spot stress.

All the different procedures for the structural stress computation are based on FE calculations. Specific techniques have been defined for structural modelling:

- according to the type of elements, FE models are thin-shell or single/multi-layer solid models; in standard fine-mesh models, elements have just corner nodes, while in relative-coarse mesh models, higher-order elements are used;
- according to the geometric and mechanics accuracy, weld is included or not − so deferring, in the latter case, to a "substitute hot spot" any reference for subsequent stress manipulations.

Fig. 2 shows a relative-coarse mesh FE model suitable for σ_{hs} extrapolation according to common standard. From a practical point of view, procedures based on low-density mesh shell-element FE models are to be preferred, provided that their accuracy and consistency is checked by comparison with solid-element models. In effect, when a solid-element FE model is employed, weld is easily possible, and the stress field both close to the weld and on the overall structural joint can be investigated with a high degree of precision.

The notch stress approach

In the "notch stress approach" the notch stress σ_k may be defined as the total local stress at the root of a notch, taking into account the stress concentration caused by both the component geometry and the local notch. In the notch-stress approach ($s = \sigma_k$), the elastic microstructural notch support hypothesis is accounted by a proper manipulation of the elastic stresses.

Fatigue-effective notch stress is calculated by averaging the elastic stresses obtained on a small area in front of the notch site. Methods for lowering the theoretical FE-calculated elastic stress at the notch are of major concern in the more recent investigations. The following methods (Radaj *et al.*, 2006) are regarded as sound reference (see Fig. 3):

- the critical distance approach, where the fatigue effective notch stress is calculated at a proper critical depth below the notch root surface; the critical depth a^* depends on the ultimate tensile strength of the material;
- the high stressed volume approach, which relates the fatigue effective notch stress to the stress gradient perpendicular to the notch surface; in details, an average elastic stress is calculated in the region $V_{90\%}$ when 90 % of the maximum notch stress is exceeded;
- the fictitious notch rounding approach states that the fatigue effective notch stress may be derived by the FE-calculated elastic stress obtained by an analysis performed on the notch with an enlarged radius ρ_f; the enlargement of the weld toe radius avoids the averaging procedure; the value of the radius ρ_f is function of the yield limit of the material.

The result of the application of the different procedures is generally expressed in term of elastic "fatigue notch factor" K_f, which may be defined as the ratio between the fatigue-effective notch stress and the nominal stress.

Calculations are carried out by making use of finite element or boundary element methods (sub-model technique). The notch stress can be also computed by fatigue notch factor K_f together with the nominal or structural hot-spot stress. The notch-stress approach based on worst-case assumption is a sound simplified procedure for investigating the effectiveness of a fatigue design up to crack initiation. The further advantage of the effective notch approach resides in that just one *S-N* curve is sufficient for representing the fatigue properties of the base material in the HAZ, since the weld notch effects have been already included in the calculated stresses. On the other hand, the method it is not suitable for code or guideline procedures because it involves very fine FE mesh models and complications in stress calculation (the stress at weld notch is highly sensitive to mesh size).

A more straightforward (and more conservative) approach proceeding from the effective notch approach is also used. In this method the fatigue notch factor is replaced by the elastic "notch stress concentration factor" K_t, defined referring the elastic notch stress to the nominal stress. According to this simplification, the notch stress based method becomes a purely geometric stress approach, where the total geometric effects are considered simultaneously.

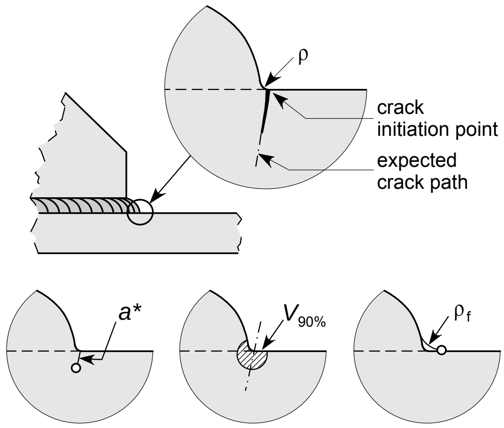

Figure 3. Different definitions for the fatigue effective notch stress.

The notch strain approach

In the "notch strain approach" the controlling parameter *s* is the strain ε. Like the notch stress approach, the notch strain approach put the fatigue life in explicitly relation to structural geometry and weld shape and to material parameters. However, unlike the former, it is able to take also into account the macrostructural support effect which arises when an appreciable local plastic deformation occurs at the notch. Basically, strain based procedures have a better formal consistency and very clearly explain the elastic-plastic mechanism which takes place around the notch root, i.e. the interaction between the notch plastic zone and the elastic surrounding material, which constraints the deformations inside of the plastic zone.

To carry out a notch-strain based fatigue assessment, the elastic-plastic strain *S*-*N* curve is necessary, along with the cyclic stress-strain curve. The cyclic stress-strain behaviour at the notch of a structural component is obtained by cyclic strain-controlled tests performed on a small unnotched comparison specimen. Fatigue life of the structural component is therefore closely connected with that of the specimen. It is widely accepted to express the cyclic stress-strain curve according to the Ramberg and Osgood formulation:

$$\varepsilon_{\mathrm{a,k}} = \varepsilon_{\mathrm{a,k}}^{\mathrm{el}} + \varepsilon_{\mathrm{a,k}}^{\mathrm{pl}} = \frac{\sigma_{\mathrm{a,k}}}{E} + \left(\frac{\sigma_{\mathrm{a,k}}}{K'}\right)^{1/n'} \tag{1}$$

where $\varepsilon_{\mathrm{a,k}}$ is the total local strain amplitude at the notch in a cycle (the sum of the elastic and plastic components), $\sigma_{\mathrm{a,k}}$ the local stress amplitude at the notch, *E* is the elastic modulus and *K*' and *n*' are material parameters. The "strain *S*-*N* curve" (also know as ε-*N* curve) is a four parameter relationship which may be written according to Manson and Coffin as:

$$\varepsilon_{\mathrm{a}} = \varepsilon_{\mathrm{a}}^{\mathrm{el}} + \varepsilon_{\mathrm{a}}^{\mathrm{pl}} = \frac{\sigma_{\mathrm{f}}'}{E}\left(2N\right)^{b} + \varepsilon_{\mathrm{f}}'\left(2N\right)^{c} \tag{2}$$

where ε_f' is the fatigue ductility coefficient, σ_f' the fatigue strength coefficient, N is the number of cycles to crack initiation and the two parameters b and c are called fatigue strength and fatigue ductility exponents respectively. An example of ε-*N* curve suitable for low to high fatigue calculations is shown in Fig. 4.

It was proved by measurements that in plate-type structures Neuber's rule may be appropriate to convert an elastically computed stress-strain pair into the corresponding actual stress-strain pair. In fact, in thin plates in tension, the material outside the elastic-plastic zone, due to the small thickness, is not efficient in restraining the deformation at the notch tip, so, the local deformation is governed by the elastic-plastic field inside of such zone. This is the base of the Neuber's rule, by which the conversion from the elastically computed to the actual elastic-plastic stress-strain field is given by the simultaneous solution of Eq. 1 and the following equation:

$$\varepsilon_{a,k}\sigma_{a,k} = K_t^2 \frac{\sigma_{a,n}^2}{E} = \frac{K_t^2}{K_{hs}^2}\frac{\sigma_{a,hs}^2}{E} \tag{3}$$

where the indexes n and hs mean that σ_a is obtained as a nominal or hot-spot stress respectively. The strain $\varepsilon_{a,k}$ obtained according to such an hypothesis is much larger than that obtained in thick plates, when the relationship between elastically computed and actual stress-strain field at the notch tip is linear. In Fig. 5 the conversion from an elastically computed nominal pair (point *A*) to the actual notch pair is shown according to both the linear law (point *C*) and the Neuber's rule (point *D*), while point *B* is the fictitious notch elastic stress and strain pair. The cyclic stress-strain curve and the ε-*N* curve of Fig. 4 and 5 are given for an unalloyed steel with ultimate tensile strength σ_u = 490 MPa and elastic modulus E = 206 GPa.

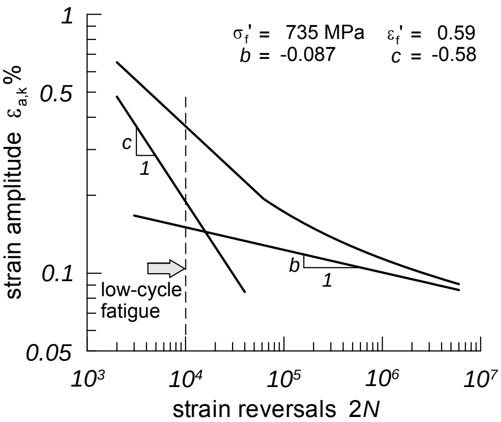

Figure 4. Strain *S-N* curve.

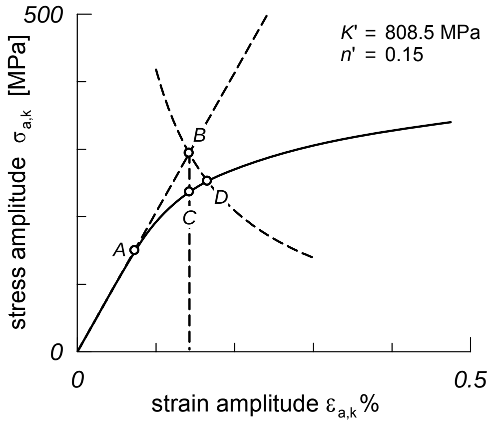

Figure 5. Notch-strain determination.

The notch-strain based assessment shows the same advantages discussed in relation to the stress-based approach. However, a number of arguments give evidence of difficulties in applying the notch strain approach, such as the inhomogeneity in material and the difficulties in defining suitable models to include the micro- and macro-structural notch support effects.

The low-cycle fatigue

Fatigue crack growth follows different laws depending on the level of stress reversals. A distinction is often made between low-cycle fatigue range and high-cycle fatigue range but the definition of the regions is somewhat arbitrary. Low-cycle fatigue is normally characterised by large strains in the plastic range with failure occurring after relative few load cycles (in the order of 10^4 cycles, see Fig. 4). Tensile strength and the ductility of the material are important parameters for the fatigue strength of components exposed to low-cycle fatigue. The endurance of the components in high-cycle fatigue is in the order of several millions of load cycles; high-cycle fatigue is generally dominated by stresses in structures which are within the elastic stress ranges. Local geometry and initial defects are important parameters in the high-cycle fatigue range.

When applied to high-cycle fatigue, notch strain and notch stress based approaches are interchangeable, as the elastic strain component of the strain *S-N* curve prevails on the plastic strain component and uniformly repeated stress cycles entail uniformly repeated strain cycles. On the other hand, the fatigue life of a structural component in the medium to low-cycle range is properly performed by the notch strain approach, on condition that investigations are aimed to ascertain the endurable strains referred to just the technical crack initiation. However, the crack initiation life according to the notch strain or stress approach and the crack propagation life according to the fracture mechanics approach may be combined to obtain the total life. In the strain-based approach residual stresses and mean stresses may be properly taken into account, along with cyclic relaxation.

The stress multiaxiality

Another open question is how to deal with multiaxial stress-strain conditions, arising when components are subjected simultaneously to multiple loads in several directions. With reference to the load carrying penetrating fillet and butt welds, the biaxial-stress hypothesis represents the most general approach for fatigue assessments. Seam welds may be studied under the assumption of a pure membrane stress field when the weld line is loaded by biaxial structural stresses with oblique principal directions. The principal stresses are substituted by a tensile stress $\sigma_\perp$ acting normally to the weld and the weld parallel shear stress $\tau_\parallel$, while the weld parallel normal stress $\sigma_{\parallel}$ is disregarded as it is generally not fatigue relevant. When the loads are applied in such a way that the principal stress orientation in the member does not vary with time, the two interacting stress amplitudes $\tau_{a\parallel}$ and $\sigma_{a\perp}$ are in phase (proportional loading), otherwise they are out of phase (non-proportional loading).

Different methods are applied within the local approaches to consider the case of proportional biaxial variable-amplitude loading. In the multiaxial approach for brittle material, the fatigue damage is calculated by referring to the maximum principal stress (MPS). So, the fatigue effective equivalent stress and the fatigue check criterion are defined as:

$$\sigma_{a,eq} = \sigma_{a,MPS} = \frac{1}{2}\left(\sigma_{a\perp} + \sqrt{\sigma_{a\perp}^2 + 4\tau_{a\parallel}^2}\right) \quad \text{and} \quad D \le D_{per} \tag{4}$$

where D_{per} is the permissible damage sum. The principal stress to be considered should be approximately in line with the perpendicular to the weld toe, i.e. within a deviation of $\beta = \pm 60°$ (see Fig. 6).

As regards ductile materials, it is suggested to make reference to one of the following approaches: the criterion of the distortion strain energy (DSE, generally referred to as the von Mises yield criterion) and the criterion of the maximum shear stress (MSS, also known as the Tresca criterion). In both cases, the fatigue effective equivalent stress and the fatigue check criterion are expressed by:

$$\sigma_{a,eq} = \sigma_{a,DSE/MSS} = \sqrt{\sigma_{a\perp}^2 + \alpha\tau_{a\parallel}^2} \quad \text{and} \quad D \le D_{per} \tag{5}$$

where $\alpha_{DSE} = 3$ and $\alpha_{MSS} = 4$ for the Von Mises and the Tresca criterion respectively.

Another approach applied in the case of proportional biaxial variable-amplitude loading considers a linear superposition of the damages caused by the two stresses acting independently; the total damage is so calculated as the sum of $D(\sigma_{a\perp})$ and $D(\tau_{a\parallel})$ as:

$$D = D(\sigma_{a\perp}) + D(\tau_{a\parallel}) = \left(\frac{\sigma_{a\perp,E}}{\sigma_{A\perp}}\right)^3 + \left(\frac{\tau_{a\parallel,E}}{\tau_{A\parallel}}\right)^5 \le D_{per} \tag{6}$$

where the index E refers to the equivalent (normal or shear) stress amplitude, that is the constant amplitude stress which is equivalent in terms of fatigue damage to the variable amplitude loading under study, at the same number of cycles N_L; and the index A refers to the permissible (normal or shear) stress amplitude at the same number of cycles N_L (see Fig. 7).

All the presented approaches are just extensions of multiaxial yield criteria. On the contrary, the "critical plane criterion" (Bäckström, 2003) is based on the observation that the critical shear stress in a shear plane is linearly dependent to the maximum normal stress in the

considered plane. To apply such an approach it is necessary to evaluate the maximum critical shear stress among the values obtained for a certain number of planes parallel to the weld line.
In all the above mentioned criteria, the cumulative damage is calculated according to the set of *S*-*N* curves defined for uniaxial loading (a proper correction may be applied to the fatigue class *FAT* value). The value of the permissible damage sum D_{per} is generally limited to 1.0 or 0.5 for steel structures (Hobbacher, 2007).
The fatigue effective equivalent stress amplitude calculated according to Eq. (4) and Eq. (5) is suitable for structural and notch stress approaches. The different stress components at the weld toe should be evaluated with separate extrapolation to the hot spot. In the notch-strain approach (low-cycle fatigue) the equivalent strain amplitude is calculated on the basis of the octahedral shear strain criterion in plane stress or in plane strain conditions.
The IIW (Hobbacher, 2007) suggests to disregard the equivalent shear stress amplitude if it is lesser than 15 % of the equivalent normal stress amplitude, or if the respective damages are in the ratio of 10 % or lesser. In the case of proportional loading, it is also recommended to use the maximum principal stress approach. In the case of non-proportional loading, it is proposed to separately calculate the damage sums of normal and shear stresses which are to be finally added as in Eq. (6).
The point of view of the classification societies is not univocal. For steel structures, DNV (2005) prescribes to calculate the cumulative damage on the basis of the maximum principal stress approach. In general, DNV states that the fatigue relevant stress is the structural principal stress at the hot spot in a sector of $\beta = \pm 45°$ from the normal to the weld line, calculated by linear extrapolation of the individual stress components of the plane stress tensor. For holes with edge reinforcement, the effects of both the structural stress parallel and normal to the weld should be considered. For each of them, a proper value of the weld stress concentration factor K_w is proposed for shifting from structural to notch stresses ($K_w = 1.5$ for the normal stress and $K_w = 1.0$ for the parallel stress). No indications are given on the method for calculating the total damage caused by the two stresses.
In the guidelines for direct calculation given by LR (2001) it is suggested to calculate the fatigue relevant stresses with fine mesh models based on shell elements. At the plating free edge in way of a hole an alternative procedure is proposed, which contemplates the use of a rod element of small nominal area incorporated just at the plating edge. The same approach is also applied, with due attention, to holes with an edge reinforcement, and the axial stress read

out on the bar positioned along the weld toe line (the parallel stress $\sigma_{\parallel}$) is considered as the fatigue relevant stress.

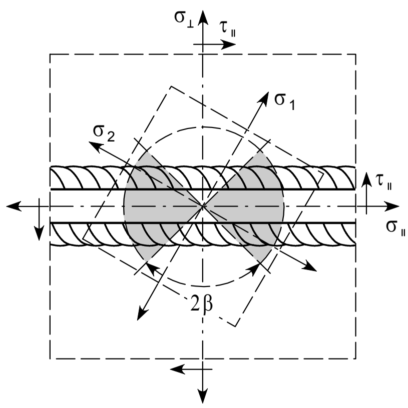

Figure 6. Biaxial oblique loading.

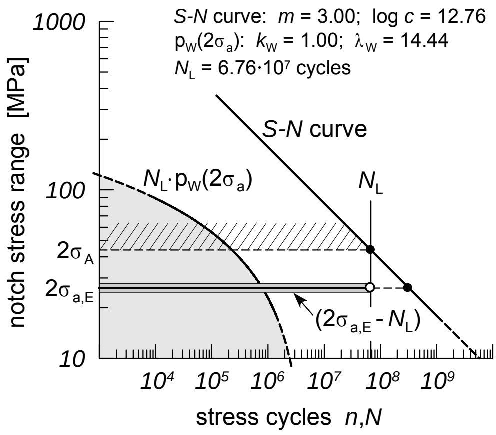

Figure 7. Equivalent and permissible stresses.

The evaluation of the cumulative damage

A direct load analysis for fatigue assessment may be carried out by making reference to a long term distribution of the wave loads. Alternatively, a simplified statistical approach may be implemented according to the Weibull probability density function $p_W(2s_a)$ – where s_a is a strain or stress amplitude. Once a significant value of s_a has been calculated together with its probability of exceedance $Q(2s_a)$, the whole load history may be approximate by the product $N_L\, p_W(s_a)$, where N_L is the total number of cycles experienced by the ship.

THE CASE STUDY

The case study concerns the fatigue analysis of the critical structural details at the openings which, vertically lined up on a longitudinal bulkhead, lead from each deck to a lift and stairs trunk (Fig. 8). At the corners of the opening high stress concentrations arise due to the high shear deformation of the bulkhead caused by the bending of the ship hull in waves. At the opening corners a biaxial plane-stress state takes place, associated to a proportional variable-amplitude loading.

Solutions aimed to the reduction of the stress level in way of the openings have been studied by focussing on a new design of the opening corners. Analyses have been carried out by making use of FE models with a mesh refinement at the opening edges. Two studies have been performed and results have been compared on the basis of the stresses caused by the maximum hogging of the ship, as the stress-strain field on the detail under consideration is governed by the hull girder deformation.

In the first study, the relation has been analysed between the corner radius r of the opening and the level of the local stresses which arise along the weld line between the bulkhead plate and the door frame (Fig. 8, "base solution"). Analyses show that the stresses parallel to the weld line according to LR (2001) are acceptable only if the corner radius is very large (i.e., $\sigma_{\|}/\sigma_{per} = 1.10$ for $r = 250$ mm), resulting in a configuration incompatible with the casing.

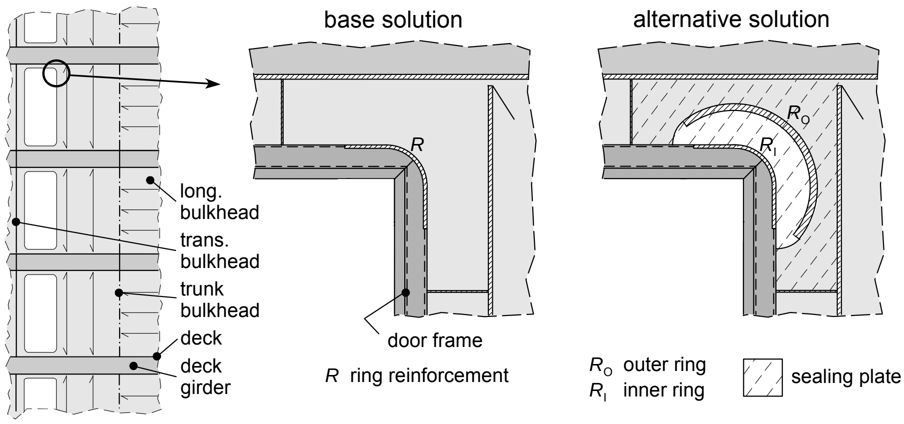

Figure 8. The opening lay-out and the structural detail.

Therefore, a second study has been performed on a "alternative solution" (Fig. 8) where, according to the standard of casing lay-out, the corner radius on the door frame is fixed to 100 mm and, in order to control local stresses on the primary supporting members, the bulkhead plate cut-out at the opening corner takes a new design with an outer corner radius of 200 mm. The gap between the two ring reinforcements (the inner ring R_I on the door frame and the outer ring R_O on the bulkhead plating edge) is sealed with a small thickness plate just to restore the *A*-class fire barrier. Such an alternative solution for the structural detail has been studied in two configurations: door frame with beam and stanchion joined with an outer rounded edge (Fig. 9, type *A*) and door frame with right-angle corners (type *B*).

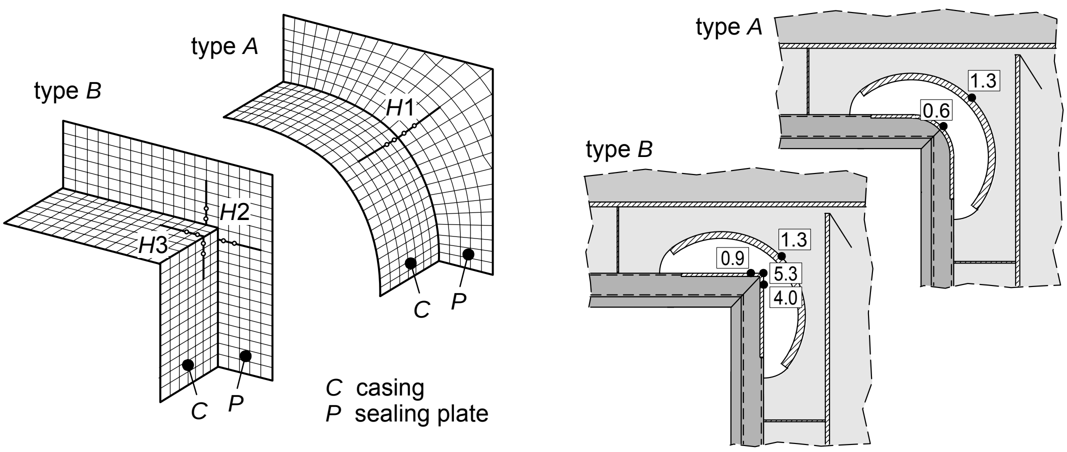

Figure 9. Hot-spot locations and calculated fatigue lives.

Maximum stresses calculated on both *A*- and *B*-type configuration are much below the highest stress measured on the base configuration with $r = 100$ mm, in the ratio of about 1:2. Since in the *A*- and *B*-type structural details peak stresses are very similar (i.e., in the ratio of 1:1.2), a complete fatigue analysis on all the hot spots of the two configurations has been carried out.

Linear fatigue analysis according to the structural-stress approach

Stresses on the critical areas are mainly caused by the vertical deformation of the hull girder and local loads have not effects on the local stresses. The extreme conditions for determining the maximum stress range $2\sigma_{a,max}$ on the hot spots are the maximum and minimum hogging of the ship hull, and the probability of exceedance of $2\sigma_{a,max}$ is $Q(2\sigma_{a,max}) = 1/N_L$. Each structural configuration has been analysed by using a shell-element FE model with fine mesh in the hot-spot area (8-node elements with side length equals to the plate thickness *t*) and coarse mesh outside. Elements have both membrane and bending capability. The welds are not modelled. Structural members are idealised, i.e. welded joints are perfectly aligned. No submodels have been created and all the FE analyses have been performed on the entire hull.

The most critical hot spots have been located on the basis of the von Mises stresses. Four different hot-spot locations have been defined:

- *H*0: weld line between ring reinforcement and bulkhead plate;
- *H*1: weld line between door frame and sealing plate in the *A*-type detail configuration;
- *H*2: weld line between door frame and sealing plate in the *B*-type detail configuration;
- *H*3: weld line between the stanchion and beam of the casing in the *B*-type detail.

The single components of the stress tensor have been linearly extrapolated to the hot spot according to procedure shown in Fig. 2. Then, the principal stresses have been calculated by the matrix diagonalization of the local stress tensor. The so obtained structural stress pair (σ_1, σ_2) has been transformed in the ($\sigma_\perp$, $\sigma_\parallel$) pair according to the DNV procedure. It has been found that in most hot spots parallel stress is greater than normal stress. DNV stress concentration factors have been applied for evaluating the notch stresses to be used in accordance to the *S*-*N* curve DNV I-b (see the *S*-*N* curve parameters in Fig. 7). N_L has been derived by an empirical formula.

The fatigue damage *D* has been evaluated by summing up $D(\sigma_\perp)$ and $D(\sigma_\parallel)$ to obtain the total damage. The *H*0, *H*2 and *H*3 hot spots have been confirmed to be very critical, with the lower fatigue lives on the side of the bulkhead plate for *H*0, on the side of the sealing plate for *H*1 and on the side of the casing stanchion for *H*3 (Fig. 9).

The hot spots at the weld lines along the radiused hole-edges (i.e., the *H*0 and *H*1 locations) have been also analysed according to the LR guidelines. The practical procedure is based on the comparison between the maximum FE calculated $\sigma_\parallel$ stress and the allowable stress. The analysed hot spots are all positively verified, in contrast with the DNV outcomes. The DNV and LR procedures are in agreement if the only $\sigma_{\parallel}$ are considered in the DNV approach.

In order to check the applied procedures, all the hot spots have been also analysed according to different biaxial criteria. The effective equivalent structural stresses $\sigma_{a,eq}$ at the hot spots have been evaluated according to:

- the maximum principal stress approach of Eq. (4),
- the criterion of the distortion strain energy of Eq. (5) with $\alpha = 3$,
- the criterion of maximum shear stress of Eq. (5) with $\alpha = 4$.

Moreover, the approach of Eq. (6) based on the linear superposition of $D(\sigma_{a}\perp)$ and $D(\tau_{a\parallel})$ has been also applied. The approaches have been compared in terms of fatigue life to the approach developed according to DNV (i.e., linear superposition of $D(\sigma_{a}\perp)$ and $D(\sigma_{a\parallel})$). Failure criterion is based on $D > D_{per} = 1$.

Results give evidence of a good agreement between the different procedures, with a clear indication on the real capability of the analysed structural details.

Non-linear fatigue analysis according to the notch-strain approach

A further fatigue check has been developed on the basis of the notch-strain approach in order to evaluate the role played by the high-stress cycles measured on most of the hot spots.

The parameters of the applied cyclic stress-strain curve according to the Ramberg and Osgood formulation are shown in Fig. 5, and the parameters of the reference elastic-plastic ε-N curve according to Manson and Coffin are shown in Fig 4. Fatigue damage has been evaluated on the basis of the linear Miner's rule. The strain reversals load history has been derived from the Weibull-based stress-range load history by simple statistical considerations. In both cases, an endurance limit at $5 \cdot 10^7$ cycles has been considered.

The non-linear fatigue analysis has been carried out on the *H*0 hot spot, as it has been considered the most important part of the structural detail. Since FE calculated stresses are basically structural stresses, a proper notch stress concentration factor $K_t = 1.96$ for cruciform joints has been considered from Laurence *et al.* (1981) in Radaj *et al.* (2006). The cumulative damage calculated according to the elastic-plastic approach is very high, more critical than that obtained by the fully elastic approach developed on the same load history and with reference to the elastic part of the same ε-N curve. The general agreement between the two approaches confirms the necessity to modify the design of the structural detail under consideration.

Moreover, the relationship between the hot-spot stress amplitude $\sigma_{a,hs}$ and the fatigue damage D on the $H0$ hot spot has been evaluated, in order to identify the limit hot-spot stress amplitude $\sigma_{a,hs,lim}$ to which the fatigue design should be aimed. The $\sigma_{a,hs,lim}$-D relationship has been calculated according to both the elastic and the elastic-plastic approach. Comparison of the two curves shows that:

- when the hot-spot stress remains below about 80 % of the yield stress of the material, the linear elastic approach leads to design on the safe side;
- the hot-spot stress for which D = 1 shows a very slight difference between the two approaches, with a +6 % according to the elastic-plastic approach.

On the basis of this consideration, one may infer that the design of the new structural detail may be suitably performed by making use of the less time consuming method, since it is on the safe side if the permissible damage is $D \leq 1$. In order to design a safe structural detail in relation to the $H0$ hot spot, the target of designer should be the containment of the hot-spot stress below about 75 % of the yield stress, as for larger stress values both approaches foreseen a fatigue failure within the ship's design life.

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