DETERMINATION OF MOORING LINES CONFIGURATION SUBMITTED TO CONCENTRATED LOADS, CURRENT PROFILE AND ON IRREGULAR SEABED.

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ABSTRACT

The oil production in deepwater demands a previous knowledge of the floating units behavior when anchored under a specific environmental conditions and, thus, the programs of numerical simulations are fundamental tools. In this paper is shown a validated model to calculate the catenary static configuration, non-rigid to flexion and submitted to current profile in line plane direction and its own weight. There is also a consideration of soil bathymetry and horizontal or slanting plane, where line can lay down (conventional configuration) or be just suspended (taut-leg configuration). All the developed procedures are implemented in a program called CATENA and the results obtained, as well as the comparison to different programs that use other methods to calculate catenary configuration, are presented in the end. The main utility of CATENA is the possibility to determine the line configuration by many parameters, as geometrical or mechanical. In CATENA, is possible to give one of these referred parameters to determinate lines configuration, that can be horizontal, vertical or strain forces on fairlead or on anchor, mooring radius (the horizontal distance between anchor and fairlead), the angle between horizontal and strain force on fairlead and on anchor or the length of backed or suspended cable. The output data are these entire characteristics and line configuration. This program is a worthwhile tool for naval engineering due to its particular features that will be presented and the good results obtained in comparison to other methods. This work was developed with ANP (Agência Nacional do Petróleo - Brazil) resource.

INTRODUCTION

The main idea to create a program which could calculate cable configurations was brought about by the lack of numerical simulators in naval and offshore engineering. Moreover, CATENA was developed to cover since homogeneous lines configuration up to heterogeneous ones, with buoy in backed or suspended cable. A method to consider a buoy in backed cable allows the existence of two rested segments: one between anchor and buoy and other between buoy and fairlead. To do so, was maintained the same equations proposed in CATENA old version [2, 4], which did not consider current profile acting on cable. At the same time, was improved the calculus method used before to a faster one, that permits achieving the right configuration in less time and number of iterations needed.

The soil bathymetry was another requirement to be considered. Using a simple archive with seabed points coordinates, CATENA can find out the correct line configuration in a good time, given one of those parameters referred above, depending on just the quantity of segments in the line to be calculated. The seabed is constructed by linear interpolation between given points. Therefore, the more points given, better soil discretization becomes.

The last issue reported in this paper cares about the new formulation obtained to consider current profile. It is very close to that formulation used herein (adapted from CATENA old version [2, 4]), but adding a possibility of change in horizontal force along whole suspended line, submitted to current loads in its plane direction.

CATENARY EQUATION

As commented above, the catenary equation that considers a possibility of change in horizontal force is very close to other formulation where it is not considered. In fact, the last case is a particular one of the first. Thus, it will be shown here the generic case, varying both horizontal and vertical forces in the line plane and then we can obtain the specific one.

Let **H** be the horizontal force and **V** the vertical one at the lower end of a suspended segment, **c** and **w** the density of horizontal and vertical forces along this segment, respectively, Δ**s** the segment length, Δ**x** and Δ**y** the horizontal and vertical distance between lower and upper ends, Δ**H** and Δ**V** the horizontal and vertical forces variation between these two points and θ the angle between the line tangent at the lower end and the horizontal, as shown in Fig. 1.

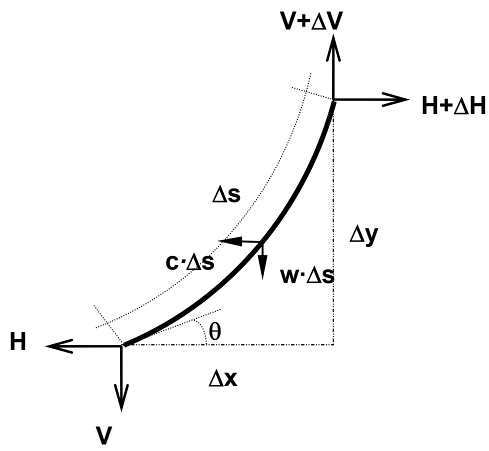

Fig. 1: Homogeneous segment submitted to horizontal and vertical forces.

Admitting small deformations, represented by ε, the line material works in elastic region and Hooke's law gives the Eq. 1 as follows:

$$\varepsilon = \frac{T}{E.A} \qquad (1)$$

where **T** is the medium strain between lower and upper ends and **E** and **A** are elasticity modulus and transversal section area of the segment, respectively. Imposing an equilibrium state, it is obtained:

$$\frac{dx}{ds} = \frac{c\,s+H_0}{EA} + \frac{c\,s+H_0}{\sqrt{(c\,s+H_0)^2+(w\,s+V_0)^2}} \quad \text{and} \quad \frac{dy}{ds} = \frac{w\,s+V_0}{EA} + \frac{w\,s+V_0}{\sqrt{(c\,s+H_0)^2+(w\,s+V_0)^2}} \qquad (2)$$

Integrating these two equations from **0** to Δ**s** = **s** (segment total length), it is obtained:

$$x(s) = \frac{s}{E.A}\left(\frac{cs}{2}+H_0\right) + \frac{c}{A^2}\left[tsup-tinf\right] + \left[\frac{H_0}{A} - c\left(\frac{B}{A}\right)\right].\ln\left|\frac{A.s+A.B+tsup}{A.B+tinf}\right| \qquad (3)$$

$$y(s) = \frac{s}{E.A}\left(\frac{ws}{2}+V_0\right) + \frac{w}{A^2}\left[tsup-tinf\right] + \left[\frac{V_0}{A} - w\left(\frac{B}{A}\right)\right].\ln\left|\frac{A.s+A.B+tsup}{A.B+tinf}\right| \qquad (4)$$

where $A = \sqrt{c^2+w^2}$, $B = \frac{H_0 c + V_0 w}{c^2+w^2}$, $tsup = \sqrt{(cs+H_0)^2 + (ws+V_0)^2}$ and $tinf = \sqrt{H_0^2 + V_0^2}$. If there is no current load, the density of horizontal force **c** will be equal to 0, reducing Eq. 3 and Eq. 4 to the old formulation adopted in the first CATENA's version, which was the source point.

Estimating c and w parameters

Consider a linear infinitesimal segment submitted to a drag force due to current and its own weight (under water), with a slope α in relation to current load, shown in Fig. 2:

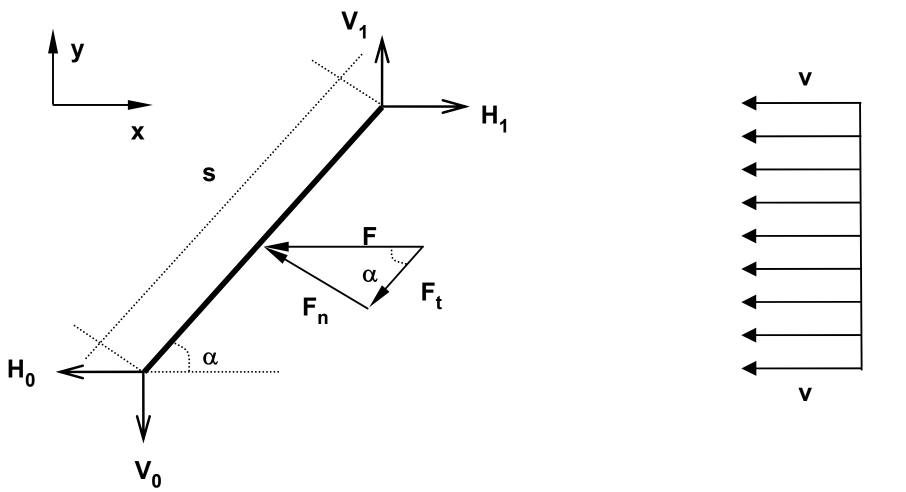

Fig. 2: Current load acting on homogeneous segment with velocity **v**.

where **v** is the current velocity (considered constant, in this case, from sea surface up to soil and positive when in opposition to axes **x**), $\vec{F}$ is the force due to current incidence and $\vec{F_n}$ and $\vec{F_t}$ are normal and tangential forces decomposed in its respective directions in relation to segment. Let us consider only the drag force $\vec{F_n}$ by linear density ($\vec{F_t}$ is too small comparing to $\vec{F_n}$) by expression [1]:

$$\frac{\vec{F_n}}{s} = \frac{1}{2}.\rho.D.C_D.\left|v.\sin(\alpha)\right|.v.\sin(\alpha) \tag{5}$$

where ρ is the water density, **D** is the hydrodynamic diameter and $\mathbf{C_D}$ is the segment drag coefficient. Therefore, if $\vec{F_n}$ is decomposed in **x** and **y** directions as showed in Fig. 2, **c** and **w** will be defined as:

$$c = \frac{\vec{F_n}}{s}.\sin(\alpha) \quad \text{and} \quad w = W - \frac{\vec{F_n}}{s}.\cos(\alpha) \tag{6}$$

where **W** is the segment linear weight density in water. The parameter **c** would be positive only when existing a current profile and equal to 0 otherwise and **w** is positive when **W** would be bigger than the drag force in **y** direction and negative otherwise.

Using this formulation proposed above, we can easily achieve in catenary configuration by an iterative method that will be described later.

For backed cable, considering μ as the friction coefficient between cable and soil, we can calculate the stress suffered by the line as illustrated in Fig. 3.

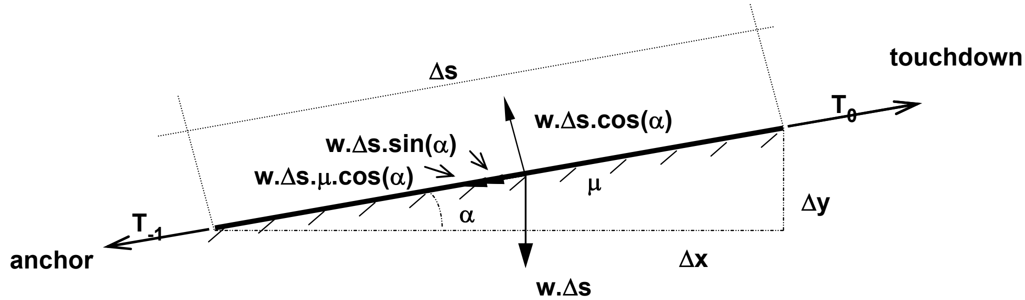

Fig. 3: Backed segment configuration in a slanting plane.

where $\mathbf{T_{-1}}$ and $\mathbf{T_0}$ are the resultant forces acting on the left and right ends, respectively. Admitting also a certain boundary conditions for the segments union (backed-suspended, backed-backed and/or suspended-suspended), it can be found any conventional or taut-leg configurations.

Boundary conditions for segments union

In suspended and backed segments, some boundary conditions must be attended to satisfy the static equilibrium. Both of them are represented in Fig. 4.

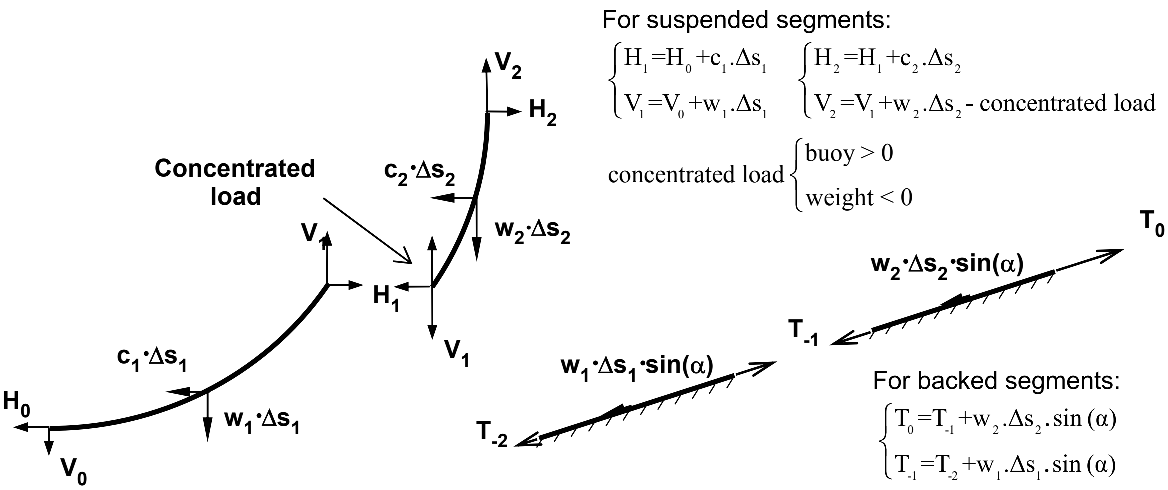

Fig. 4: Boundary conditions for segments union. Suspended (left) and backed (right) cables.

ITERATIVE METHOD TO OBTAIN CATENARY CONFIGURATION

From this moment on, it will be developed a method to calculate the catenary configuration using the equations and boundary conditions mentioned above, for cases which consider a current profile or not.

Let us admit a cable with whole characteristics defined as: number of homogeneous segments and, for each one: length (**s**), linear weight density (**W**), elastic constant (**EA**), minimum break load (**MBL**) and friction coefficient (μ); number of concentrated loads and, for each one: position in line, force (positive for buoy and negative for weight). Basically, CATENA compares the depth required by user to that obtained by each iteration, using the following objective function (**OF**):

$$OF = Y - X.\tan(\alpha) \tag{7}$$

where **Y** and **X** are the vertical and horizontal distances between fairlead and touchdown, as represented in Fig. 5.

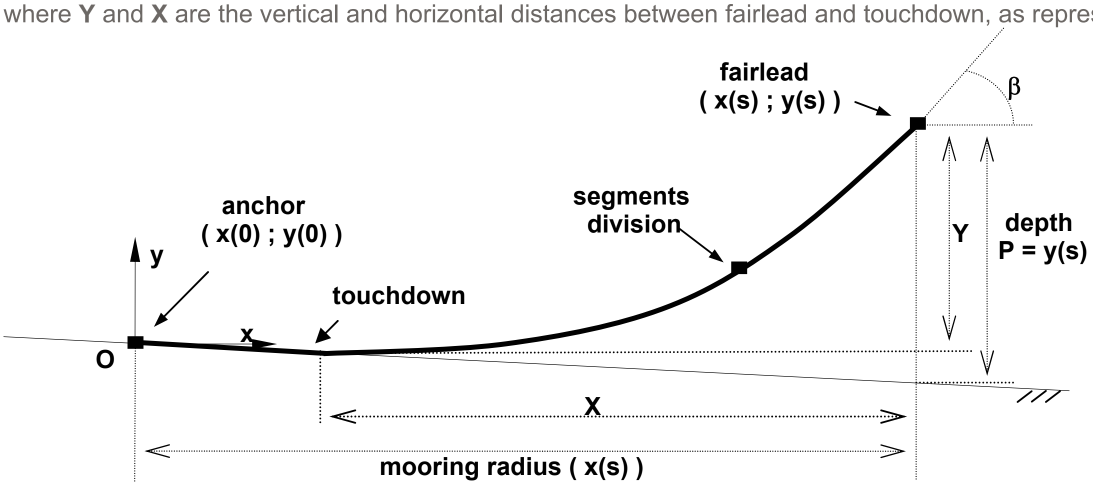

Fig. 5: Conventional mooring line representation in a slanting plane.

where **O** is the anchor position and **P** the depth under unity. For the first iteration, CATENA considers a taut-leg configuration with an angle between line and soil, in anchor, equal to 0. This could be called limit situation, in other words, it is between conventional and taut-leg configurations. Therefore, using the Eq. 3 and Eq. 4 and the boundary conditions, we will have a result that could be one of these showed in Fig. 6.

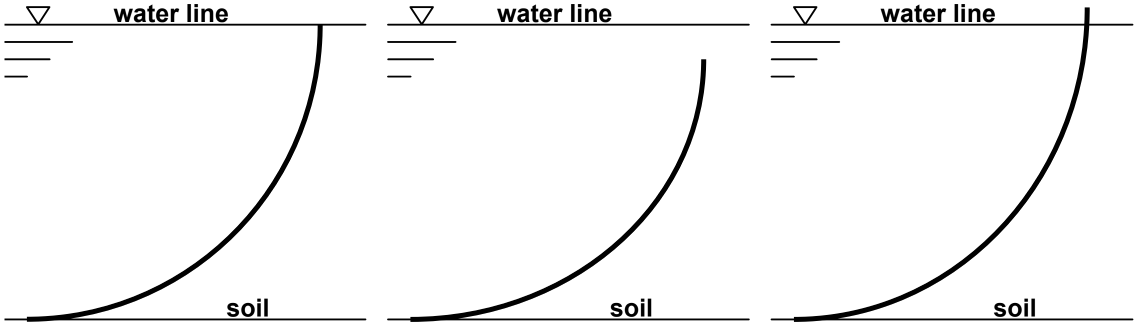

Fig. 6: Possible line configurations after first iteration by CATENA.

So, as seen in Fig. 6, if the obtained configuration is the left one, the problem is solved (**OF** = **P**). On the other hand, if it is the middle situation, configuration will be taut-leg (**OF** < **P**) and CATENA will iterate in angle between line and soil up to achieve a convergence and a configuration seemed to the left. Otherwise, the right configuration (**OF** > **P**), makes CATENA iterate in backed segment length up to a convergence and achieve a configuration seemed to the left, giving a conventional line as the result.
The method used in CATENA could be summarized in (even when current profile is considered):

1. For line's first configuration, consider **c** = **0**;
2. Estimate the vertical force on anchor by $L = H * \tan(\alpha)$;
3. Calculate the horizontal and vertical forces in each segment by conditions showed above;
4. Apply Eq. 3 and Eq. 4 and obtain **X** and **Y** as defined on Fig. 5;
5. If **OF** showed in Eq. 7 is equal to **P**, the configuration is done. If **OF** < **P**, iterate in angle α. Otherwise **OF** > **P**, iterate in the backed length segment (**b**) up to achieve a convergence.
6. For other iterations, estimate **c** and **w** parameters based on last configuration and go back to step 2 up to satisfy **OF**.

An iterative method application with buoy presence in backed segment case

When buoy is present in backed segment, the problem requires a different analysis. First, consider the line without having any concentrated load and use the steps 1 to 6 showed before. After achieving the correct configuration as showed in Fig. 6 (left), CATENA starts to iterate in whole rested segment length which has the buoy.
Now, the objective function will change from that showed in Eq. 7 to another, whose parameters to compare are the vertical forces applied on the top of the segments, one between anchor and buoy and other between buoy and touchdown, as illustrated in Fig. 7.

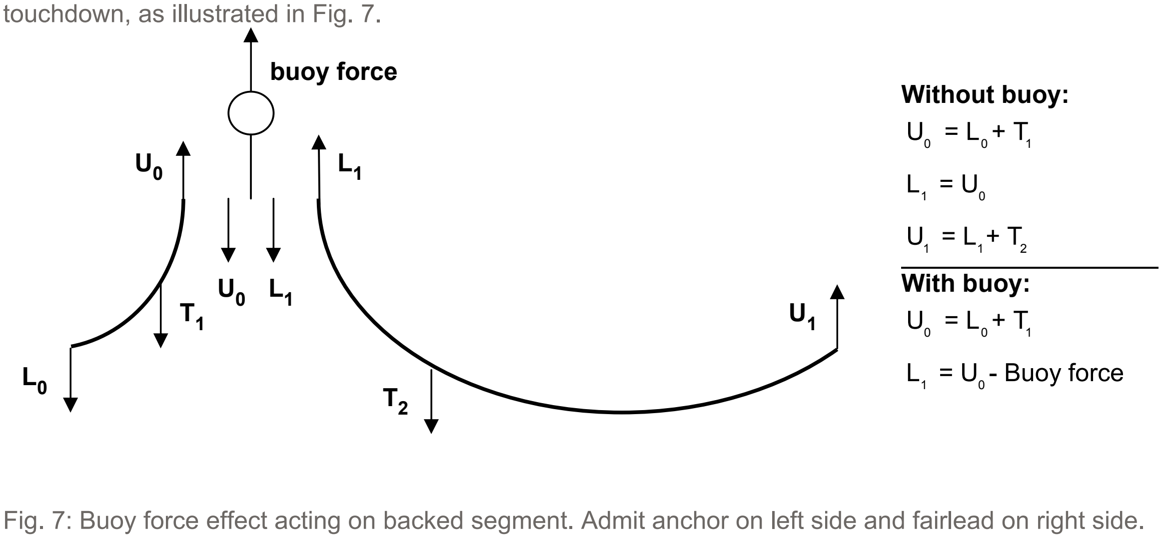

Fig. 7: Buoy force effect acting on backed segment. Admit anchor on left side and fairlead on right side.

where **L** and **U** are forces applied in the end of segments and **T** is the total weight of each one. The formulation used to calculate both segments (before and after the buoy) is the same showed in Eq. 3 and Eq. 4, but considering **c** equal to 0, because is not possible yet to consider current profile in this kind of configuration in CATENA. As commented before, the new **OF** is, now, the sum of U_0 and L_1. If this sum is less than buoy force, we have to suspend more part of the segment

after the buoy and before the touchdown. Otherwise, if the sum is greater then buoy force, we have to back more segment between buoy and touchdown.
The configuration obtained on right side of the buoy in Fig. 7, allows CATENA to calculate the segment high (in relation to soil) in buoy position and pass it as a reference to left side segment, which could calculate a new configuration of the line. In this way, iterating the backed segment on right side and checking out for every iteration **OF** value, we can found out the catenary configuration whit buoy even in rested cable. The results obtained using this method are very close to other numerical simulators.
In case of having weight instead of buoy in backed cable, there is no problem to calculate too, because the weight pulls the line down and so, we do not have to iterate in this segment.

BATHYMETRY CONSIDERATION

One of the most important considerations CATENA does is the bathymetry. Given an assemblage of points which represents the seabed of determinate area and one geometrical or mechanical parameter, as cited in abstract, it is possible to achieve a configuration considering a irregular soil.
This tool makes CATENA an important program, inasmuch as other programs hardly gives this chance to user. Using again the previous formulation and also the same method, we can easily find out a solution. In the file given to CATENA, besides the assemblage of points, it shall be given the fairlead position in the system coordinates adopted by user and the azimuthal angle (angle between the launching line and the **X** axes of global system).This angle tells us in which plane line is localized. Having this information, it is very easy to describe the soil where line is positioned by a linear interpolation between points.
The method used to obtain the line configuration, considering a specified soil, consists in analyze each soil segment as reference plane. With this plane and its respective depth, we can check out if the right solution passes through it and horizontal distance between touchdown and fairlead is inside of the interval of soil segment. If the answer is yes, the solution is found, otherwise, go to the next soil segment. It ends when solution is achieved. Fig. 8 represents, on left, a possible solution situated in segment number 3 of the soil and, on right, the right solution in this same segment.

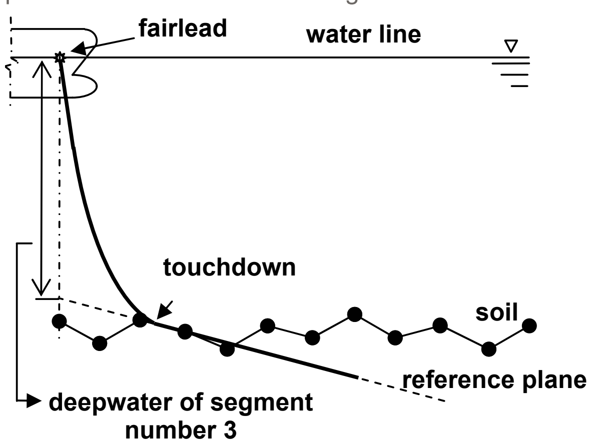

Fig.8: Analysis for a possible solution when considering bathymetry (left) and solution achieved by CATENA (right).

RESULTS AND CONCLUSION

The output data of CATENA method were compared with two other methods results: FEM [8], a method based on cable analysis using the Finite Element Method and Preadyn [3, 7], a FEM method with dynamic solutions. Some results are presented here. FEM and Preadyn were used in comparisons with no current profile and only FEM was used to compare results considering current incidence.
Table 1 shows the line properties used for comparisons.

Table 1. Line types used for comparisons.

Line type	Material	W (kN/m)	EA (kN)	MBL (kN)	C_D	D_H	μ
1	Steel	0.505	944261	8028	1.7	0.175	0.5
2	Steel	0.454	386265	7900	1.2	0.115	0.5
3	Chain	1.864	434740	8730	1.2	0.122	0.5
4	Polyester	0.050	279398	8339	1.2	0.17	0.5

Lines properties:

W: linear weight density in water **EA:** Elasticity modulus multiplied by transversal section area

MBL: Minimum break load

D_H: Hydrodynamic diameter

C_D: Drag coefficient

μ: Static friction coefficient

The following figures and tables, Fig. 9 to Fig. 11, and Table 3 to Table 8, refer to these comparisons. In comparison 1 was used a conventional heterogeneous line, in comparison 2, a taut-leg homogeneous line, in comparison 3 a homogeneous line with a buoy, in comparison 4, a homogeneous line with a buoy and a concentrated weight, in comparison 5, a homogeneous line submitted to a current and, in comparison 6, there is a demonstration of a possible solution obtained by CATENA on an irregular soil. Table 2 shows the conditions of each comparison made from 1 to 6.

Table 2. Comparisons characteristics.

Comparison	**Segments Properties**			**Depth (m)**	**Buoys (+) and Concentrated Weights (-)**			**Mooring Radius (m)**	**Horizontal Force (kN)**	**Current Velocity (m/s)**
	Number of segments in the line	Type	Length (m)		Number	Position in line	Force (kN)			
1	3	1	500	800	-	-	-	1000	-	0.0
		2	950							
		4	50							
2	1	3	1700	600	-	-	-	1580	-	0.0
3	1	3	1700	600	1	300	300	1200	-	0.0
4	1	3	1700	600	1	300	300	1200	-	0.0
					2	1500	-500			
5	1	1	1700	1000	-	-	-	-	0	3.0
6	1	1	1700	1000.36	-	-	-	-	100	0.0

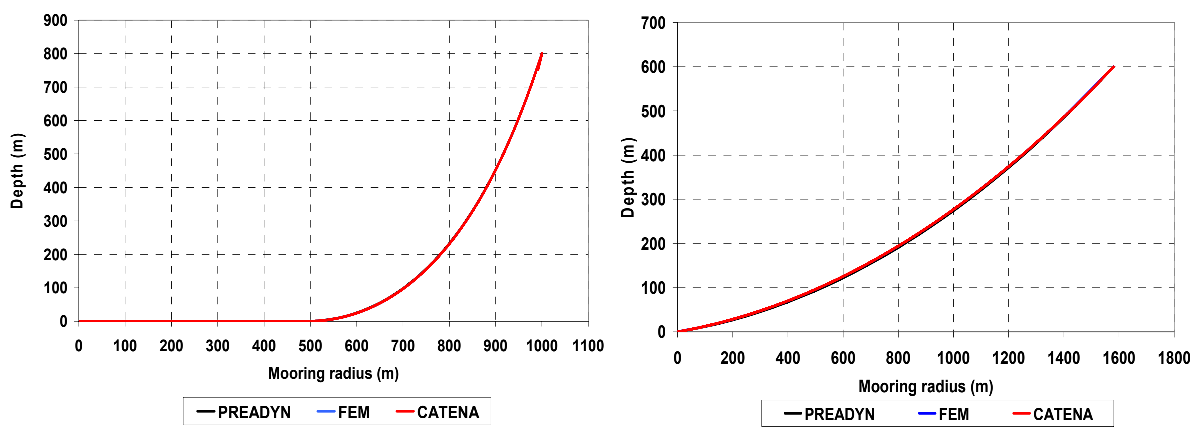

Fig. 9. Comparisons 1 (left) and 2 (right).

Table 3. Comparison 1.

	Horizontal force on fairlead (kN)	**Vertical force on fairlead (kN)**	**Vertical force on anchor (kN)**	**Horizontal force on anchor (kN)**	**Mooring radius (m)**	**Backed length (m)**	**Suspended length (m)**	**Total deformed length (m)**	**Angle on fairlead (º)**	**Angle on anchor (º)**
Catena	105.77	436.46	0.00	0.00	1000.00	498.58	1002.12	1500.70	76.38	0.00
Fem	105.76	435.16	0.00	0.00	1000.00	500.00	1000.00	1500.75	76.34	0.00
Preadyn	127.88	532.20	-12.35	127.82	1000.00	487.76	1012.24	1500.00	76.32	0.00

Table 4. Comparison 2.

	Horizontal force on fairlead (kN)	**Vertical force on fairlead (kN)**	**Vertical force on anchor (kN)**	**Horizontal force on anchor (kN)**	**Mooring radius (m)**	**Backed length (m)**	**Suspended length (m)**	**Total deformed length (m)**	**Angle on fairlead (º)**	**Angle on anchor (º)**
Catena	1541.12	1027.35	168.85	1541.12	1580.00	0.00	1706.53	1706.53	33.69	6.25
Fem	1540.97	1020.58	175.50	1540.97	1580.00	0.00	1706.51	1706.51	32.30	6.21

Preadyn	1797.97	1186.08	211.38	1797.97	1580.00	0.00	1707.59	1707.59	33.41	6.70

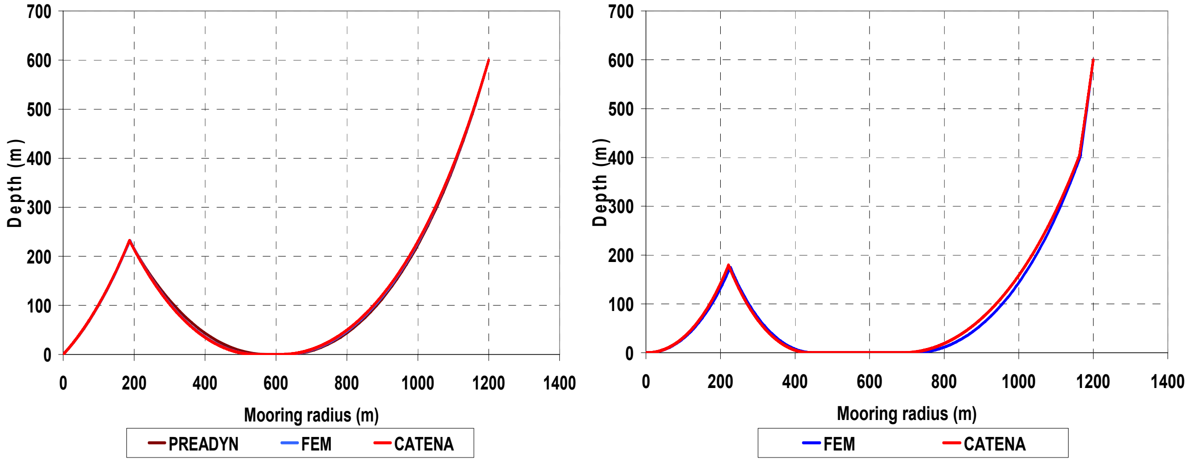

Fig. 10. Comparisons 3 (left) and 4 (right).

Table 5. Comparison 3.

	Horizontal force on fairlead (kN)	Vertical force on fairlead (kN)	Vertical force on anchor (kN)	Horizontal force on anchor (kN)	Mooring radius (m)	Backed length before the buoy(m)	Suspended length after the buoy (m)	Backed length after the buoy (m)	Total deformed length (m)	Angle on fairlead (º)	Angle on anchor (º)
Catena	180.06	448.00	124.06	155.87	1200.00	0.00	887.74	68.46	1700.99	68.10	38.52
Fem	177.07	425.85	130.42	156.94	1200.00	0.00	877.17	79.73	1701.02	67.74	39.72
Preadyn	162.20	496.14	83.59	162.20	1200.00	0.00	877.55	54.46	1699.92	68.50	38.79

Table 6. Comparison 4.

	Horizontal force on fairlead (kN)	Vertical force on fairlead (kN)	Vertical force on anchor (kN)	Horizontal force on anchor (kN)	Mooring radius (m)	Backed length before the buoy(m)	Suspended length after the buoy (m)	Backed length after the buoy (m)	Total deformed length (m)	Angle on fairlead (º)	Angle on anchor (º)
Catena	166.68	931.46	0.00	77.75	1200.00	2.97	855.16	248.66	1700.94	79.85	0.00
Fem	151.16	895.01	0.07	83.30	1200.00	0.00	834.35	268.83	1701.00	80.41	0.05

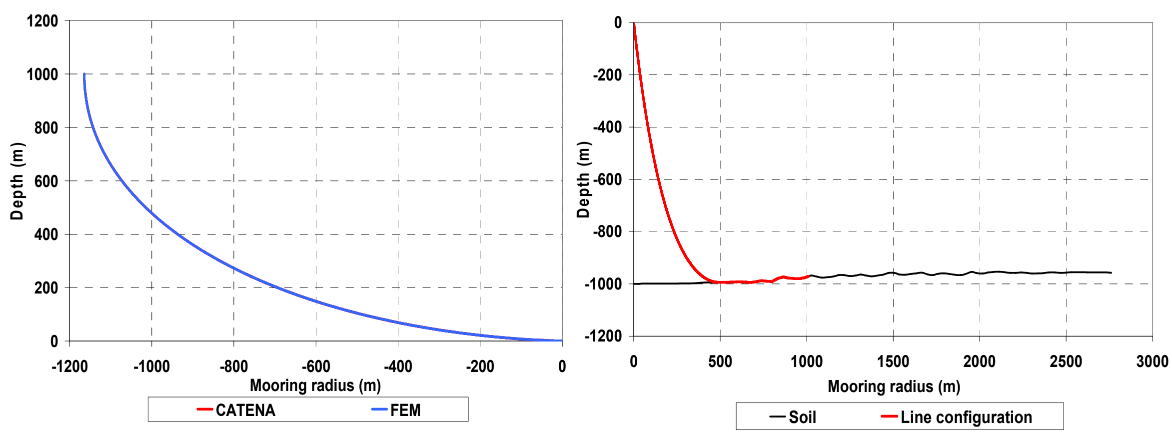

Fig. 11. Comparisons 5 (left) and 6 (right).

Table 7. Comparison 5.

	Horizontal force on fairlead	Vertical force on fairlead	Vertical force on anchor	Horizontal force on anchor	Mooring radius (m)	Backed length (m)	Suspended length (m)	Total deformed length (m)	Angle on fairlead	Angle on anchor

	(kN)	(kN)	(kN)	(kN)					(º)	(º)
Catena	0.00	1355.69	39.67	850.36	1164.42	0.00	1700.00	1701.21	90.00	177.33
Fem	0.00	1351.05	41.64	847.46	1162.95	0.00	1700.00	1701.84	90.00	177.18

Table 8. Bathymetry.

	Horizontal force on fairlead (kN)	Vertical force on fairlead (kN)	Vertical force on anchor (kN)	Horizontal force on anchor (kN)	Mooring radius (m)	Backed length (m)	Suspended length (m)	Total deformed length (m)	Angle on fairlead (º)	Angle on anchor (º)
Catena	100.00	593.34	0.00	0.00	1011.93	522.10	1178.32	1700.42	80.43	0.00

As it can be seen in all compared results above, the graphics and data are very good, even when it is compared to other methods of analysis. Although it is not showed in this paper, there are a lot of possible configurations, varying input data of those parameters presented in beginning and submitted to different situations. An attempt of developing this project is in progress and consists in adding more tools to CATENA, in order to become it a more powerful program.
In comparisons 1 to 4, FEM results may be taken as exactly. All differences are equal or lower than 5%, except for values close to zero, where absolute values are more significant and, in some cases, for backed length. CATENA takes the exact touch down point, while in FEM there is an element with one backed node and one suspended. The real touch down point is between these two nodes. Since the curvature and tension precision are obtained, there is no need to divide this element. So, the obtained backed length may be distorted. In comparison 4 there is an example of this situation.

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