

SOME CONSIDERATIONS ABOUT B-SPLINE HULL SURFACE REPRESENTATION

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ABSTRACT

B-Spline hull surface representation is here considered for panel method applications. Approximate analytical surfaces are obtained by a least square deviations method from offset table data points. Some results for a Wigley hull and for a Series 60 hull are presented. The analytical sectional curves representation agrees well with the station points up to the design draft, but if lofting qualities are to be improved other optimization techniques may be needed.

INTRODUCTION

Ship hull surfaces are traditionally represented by offset tables. Algebraic continuous representation of these surfaces meets difficulties especially regarding lofting qualities. Yet algebraic representation with a limited number of parameters would be convenient for design, analysis and construction, because:

- geometric properties of the hull could be determined more efficiently and accurately;
- geometry input for hydrostatics, hydrodynamics, structure, layout and assembly could be more easily prepared and dealt with;
- methods for estimation of these attributes could be better conditioned, resulting in faster and more precise techniques;
- due to decrease in time and increase in accuracy of the estimation methods, the search could be broadened.

It is not expected that a single algebraic representation meets all requirements for the various stages and aspects of ship hull design, analysis and construction. In fact, the design process begins with concise representations, while detailed descriptions are to be built for implementation. Also, the degree of needed accuracy increases during the design process as the number of alternatives diminishes.

B-Spline or rational Spline surfaces may be the choice for hull representations in many design phases, but still there are various well-founded possibilities regarding degree of curves, class of continuity, parameterization and weights.

It is an open task, and a huge one, to build adequate and comprehensively linked algebraic geometric representations for the various phases of a hull design. This paper addresses specifically a B-Spline representation for wave resistance and seakeeping computations using panel methods. The algebraic representation is more convenient for elaborating the panel mesh than the offsets table, but also the method itself may benefit prospectively from this continuous geometry representation.

APPROXIMATION OF A B-SPLINE SURFACE TO DATA POINTS

Let $\bar{s}^{m,n}(u, v) = \sum_{i=0}^{L_m+m-1} \sum_{j=0}^{L_n+n-1} \bar{d}_{i,j} N_i^m(u) N_j^n(v)$ be a B-Spline surface, where u and v are parameters,

$N_i^m(u)$ and $N_j^n(v)$ are B-Spline basis functions of degree m and n respectively, and $\bar{d}_{i,j}$ are vertices of the surface. Suppose that data points \bar{p}_i , $i=0, \dots, P$, are given as in an offset table. The points ideally belong to an unknown surface $\bar{p}_{IDEAL}(u, v)$. The surface $\bar{p}(u, v)$ is optimized to find the best fit to this ideal surface $\bar{p}_{IDEAL}(u)$. Suppose this approximation is a B-Spline surface, based on the knots

sequences $(u_0, \dots, u_{L_m+2m-2})$ and $(v_0, \dots, v_{L_n+2n-2})$ with the domain $[u_{m-1}, u_{L_m+m-1}] \times [v_{n-1}, v_{L_n+n-1}]$. Let $u = v_i$ and $v = v_i$ be parametric values associated to the data points \bar{p}_i . Probability distributions of the deviations of the data points from the ideal surface are assumed normal: $\exp\left(-\frac{(p_i - p_{\text{IDEAL}}(v_i, v_i))^2}{\sigma_i^2}\right)$.

Considering that all standard deviations are equal, the problem of finding a set of vertices that define the intended surface with maximum likelihood becomes the problem of minimizing $\sum_{i=0}^P (p_i - p_{\text{IDEAL}}(v_i, v_i))^2$,

$$\text{that is, of minimizing } \sum_{i=0}^P \left(\bar{p}_i - \sum_{j=0}^{L_m+m-1} \sum_{k=0}^{L_n+n-1} \bar{d}_{i,j} N_j^m(v_i) N_k^n(v_i) \right)^2.$$

Taking the derivative with respect to \bar{d} (considered as each coordinate of \bar{d}) and setting it equal zero results in this system of equations:

$$\sum_{i=0}^{L_m+m-1} \sum_{j=0}^{L_n+n-1} \bar{d}_{i,j} \sum_{k=0}^P [N_i^m(v_i) N_j^n(v_i) N_k^m(v_i) N_\ell^n(v_i)] = \sum_{i=0}^P [N_k^m(v_i) N_\ell^n(v_i) p_i] \quad (1)$$

$$k=0, \dots, L_m + m - 1, \quad \ell = 0, \dots, L_n + n - 1,$$

The above equations form a linear system with $(L_m + m) \cdot (L_n + n)$ unknowns $\bar{d}_{i,j}$, $i=0, \dots, L_m + m - 1$, $j=0, \dots, L_n + n - 1$, which hopefully leads to the intended approximate surface. This method of approximation does not consider specific lofting qualities and criteria. The class of continuity is associated with the degree of the B-Spline surface.

STATION CURVATURE ESTIMATION

The distribution of curvature of the stations may be estimated by a series of steps. The first is determining the tangent vector to the curves, followed by the normal vector, and finally the curvature:

$$\text{Tangent: } \vec{T} = \frac{\partial(\vec{s}^{m,n}(u_{\text{sta}}, v))}{\partial v} = n \sum_{i=0}^{L_m+m-1} \sum_{j=2}^{L_n+n-1} \frac{\bar{d}_{i,j} - \bar{d}_{i,j-1}}{v_{j+n-1} - v_{j-1}} N_i^m(u_{\text{sta}}) N_j^{n-1}(v)$$

$$\text{Normal: } \vec{M} \cdot \vec{T} = 0$$

$$\text{Curvature: } \kappa(u_{\text{sta}}, v) = \vec{M} \cdot \frac{\partial^2 \vec{s}^{m,n}(u_{\text{sta}}, v)}{\partial v^2}$$

$$= n(n-1) \sum_{i=0}^{L_m+m-1} \sum_{j=2}^{L_n+n-1} \vec{M} \cdot \frac{1}{v_{j+n-2} - v_{j-1}} \left(\frac{\bar{d}_{i,j} - \bar{d}_{i,j-1}}{v_{j+n-1} - v_{j-1}} - \frac{\bar{d}_{i,j-1} - \bar{d}_{i,j-2}}{v_{j+n-2} - v_{j-2}} \right) N_i^m(u_{\text{sta}}) N_j^{n-2}(v) \quad (2)$$

SOME RESULTS FOR A WIGLEY HULL

A Wigley hull with bi-parabolic representation is considered first. Despite the surface being quadratic, an approximation with a bi-cubic B-Spline will be tried, with 5 equally distributed points along the 11 stations. Parameters u and v are taken to be the longitudinal and the vertical positions, respectively. Two pieces of B-Splines are considered in each parametric direction. Figure 1¹ presents the results of the approximation.

¹ All figures at the end of the paper.

It is seen that the approximation is exact in this case. The planes with a fixed slope for the longitudinal (X) and vertical (Y) coordinates in respect to the parameters u and v just reflect the choice of parameters. The resulting hull form is effectively bi-quadratic, and the cubic coefficients properly remained zero. The resulted surface passes through all the given points.

SOME RESULTS FOR A 2D SERIES 60 STATION

Next, a Series 60 station is considered, described by 7 points. Results for a B-Spline approximation with two pieces of cubic curves are presented in figure 2. Here the flat part of the bottom has turned into a curve. Another approximation was tried, with more data points describing the station, generated on the assumption that the bottom is flat. The approximation for the increased number of data points was tentatively tried to a four piece B-Spline curve. Results are in figure 3. Now it is seen that the representations become much more similar to the intended curve.

SOME RESULTS FOR A 3D SERIES 60 HULL

Several different parameterizations were tried for the complete Series 60 hull form, the majority of which did not achieve the desired accuracy. The Series 60 hull form was a $C_b = 0.6$, with a length to beam ratio of 7.0 and a beam to draft ratio of 3.5. For convenience the draft was set equal to 1.0 and 291 offset points were used.

The most successful parameterization was based on an axis-symmetric surface. As shown in figure 4, the u and v coordinates of the axis-symmetric surface are the length of its meridian measured from the bow and the distance around the girth measured up from the keel, respectively. Parameter u was normalized with respect to the maximum length of the meridian of the axis-symmetric body and v was normalized with respect to the local girth up to the deck line (1.5 times the draft for the Series 60 hull).

Two different bi-cubic approximations for the B-Spline surface were used. The first approximation required the tangent to the hull on the keel to be horizontal; the second allowed this tangent to be free.

The global mean square distance deviations of the fitted surface from the original input offsets are 0.88% for the constrained case and 0.70% for the free case.

The parametric representation of the approximation for the coordinates is presented in figure 5. In this figure, one may notice that the x longitudinal coordinate has an almost linear dependence on the u parameter. The y vertical coordinate graphic shows an apparent flat region that corresponds to the bottom. The z lateral coordinate, which is symmetric with respect to parameter v , presents on both sides an apparent flat region that corresponds to the flat part of the side.

One isoparametric curve for $v=0.30$ is presented in figure 6. This is a diagonal cut, which presents a visual smoothness. Part of the cut is apparently flat, corresponding to crossing the flat part of the side.

The station representation and results obtained for both bi-cubic approximations are presented in figures 9a to 9h, as well as the distribution of curvatures against the contour length of the station up to the deck. The horizontal distances have been normalized by half beam. There are 6 equally spaced intervals in the u direction and 5 in the v direction. It is seen that graphically the quality of the approximations are similar. The curvature graphics show cusps at the bilge, which might be an indication that a re-parameterization of that region with shorter parameter intervals could improve the results.

Figure 7 shows the expected and the evaluated curvature distribution for station 10, where the bilge radius is 0.5. The bottom is flat for $0 \leq v \leq [(1.75 - 0.5)/(\text{girth to deck}) \cong 0.49]$, being the curvature null; the side is flat for $v > [1.0 - (1.5 - 0.5)/(\text{girth to deck}) \cong 0.67]$, being also the curvature null. For intermediate values, the curvature is expected to be $1/0.5 \cong 2$.

Figure 8 presents the tangent vector for this section. This graphic also makes clear that there are some oscillations of the originally flat parts of the bottom and side. Figure 9e helps to understand that closer to the centerline there is a slight concave curvature (negative) that and turns into a convex curvature (positive) towards to the bilge, becoming highest in the center of the bilge. Then it diminishes towards the side.

Figure 10 shows some views of a grid for the approximated hull surface, here including the bow and the stern.

ANALYSIS

It is possible to represent hull surfaces analytically by B-Spline surfaces with different degrees of freedom and parameterizations. The global deviation of the surface to the original points may not be a sufficient mean to assure the quality of the approximation. This quality must be verified in respect to the particular applications that are envisaged for the approximation. Yet, having the fitted surface RMS error been very small (0.70%), it is expected that the geometric integral properties (areas, volumes, moments of areas and volumes, centers of areas and volumes) will be good. Some other applications may require different approximation processes, where the appropriate criteria are taken into account.

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- 3 - Gallier, J. Curves and Surfaces in Geometric Modeling. Morgan Kaufmann, San Francisco, 2000.
- 4 - Todd, F. H. Series 60: Methodical Experiments with Models of Single-Screw Merchant Ships. David Taylor Model Basin, Report 1712, July/1963.

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FIGURES

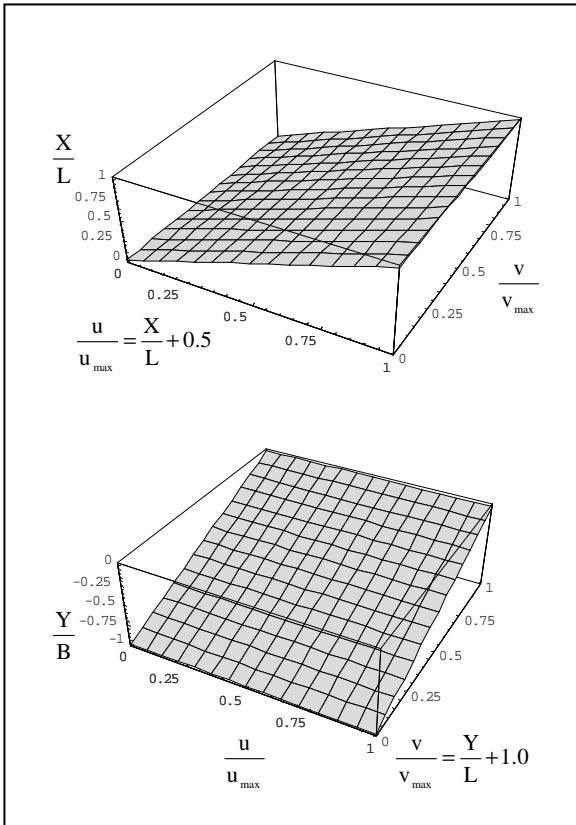


Figure 1a - Longitudinal and vertical coordinates approximations for the Wigley hull.

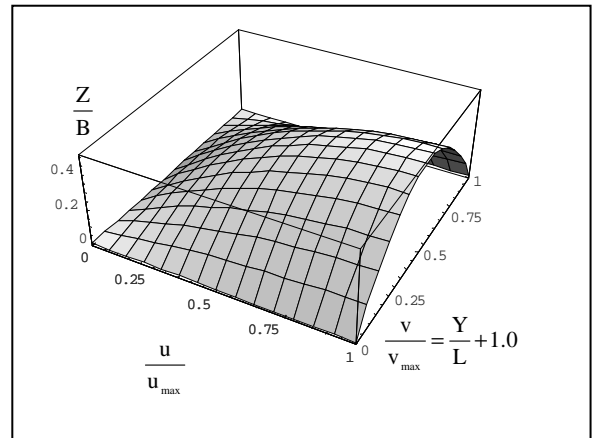


Figure 1b - Lateral coordinate approximation for the Wigley hull.

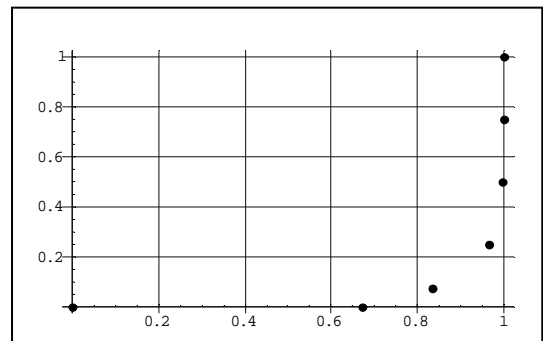


Figure 2a - Series 60 - station 9 - from offsets table - 7 data points.

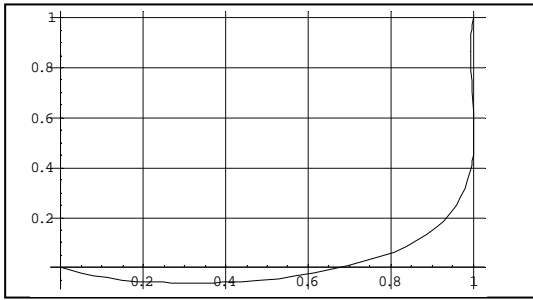


Figure 2b - Approximation for station 9; 7 data points; 2 pieces of cubic curves.

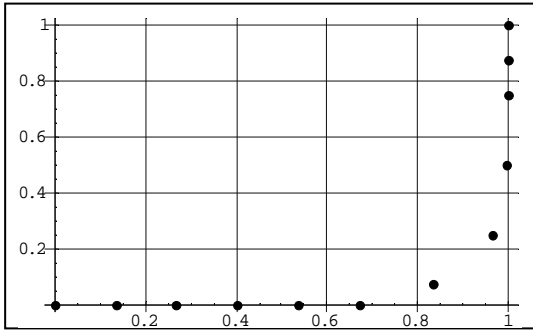


Figure 3a - Series 60 - station 9 - from offsets table - 12 data points.

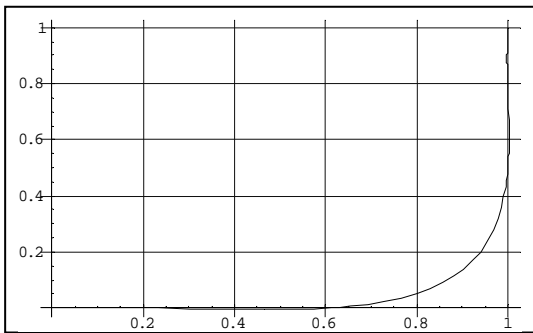


Figure 3b - Approximation for station 9.

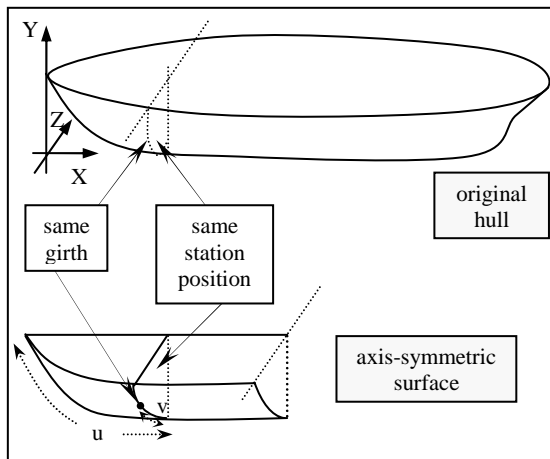


Figure 4 - Hull parameterization.

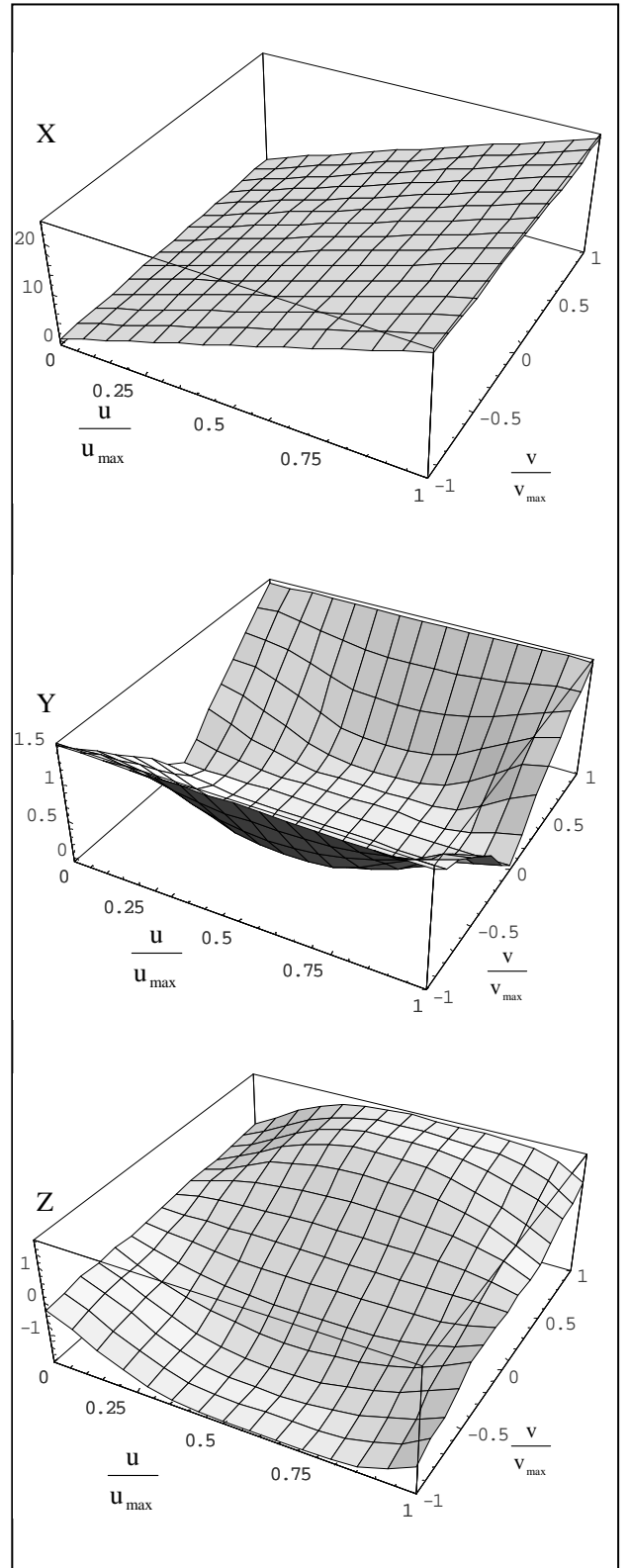


Figure 5 - Series 60 3D hull symmetric representation.

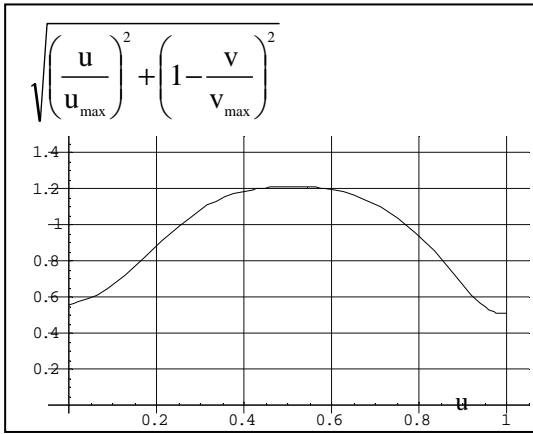


Figure 6 - Isoparametric curves of the hull at parameter values $v=0.30$.

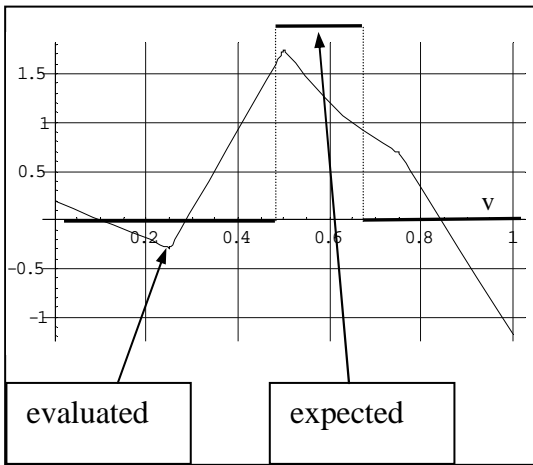


Figure 7 - Expected and evaluated curvatures \tilde{n} Series 60 - symmetric - Station 10.

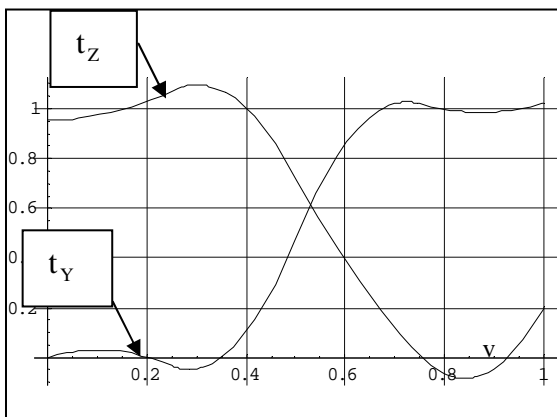


Figure 8 - Evaluated tangent vector \tilde{n} Series 60 - symmetric - Station 10.

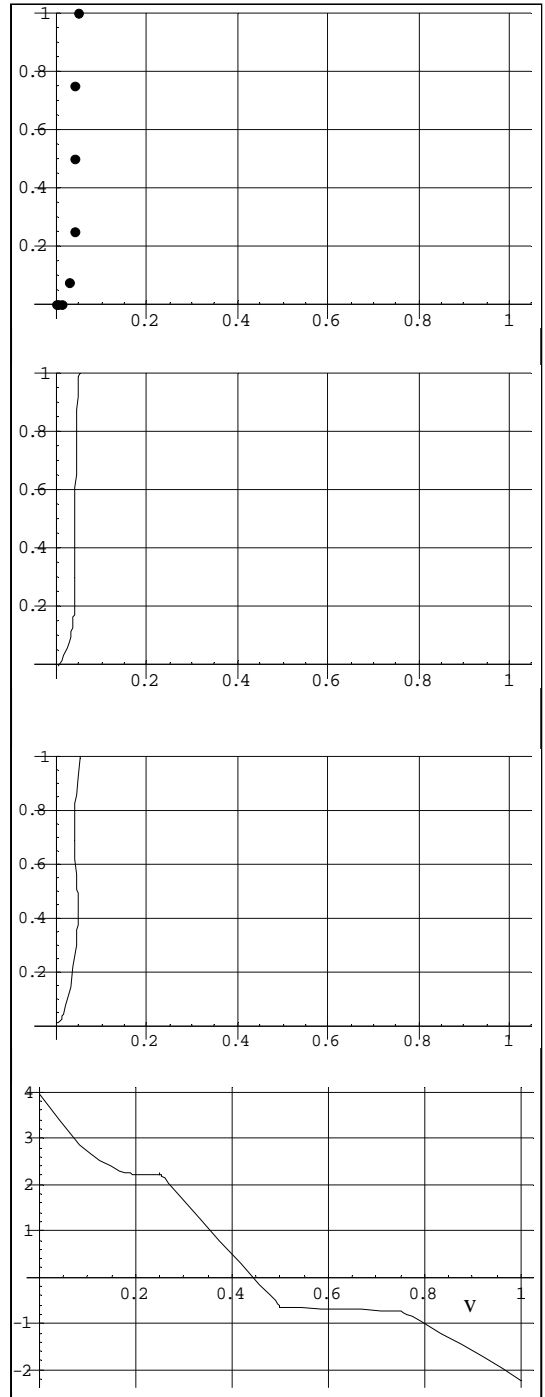


Figure 9a - Series 60 - Station 0.5
Points from the offsets table (upper), bi-cubic approximation without tangent constraint on the keel (middle upper); with constraint (middle lower); curvature (lower) versus arc length

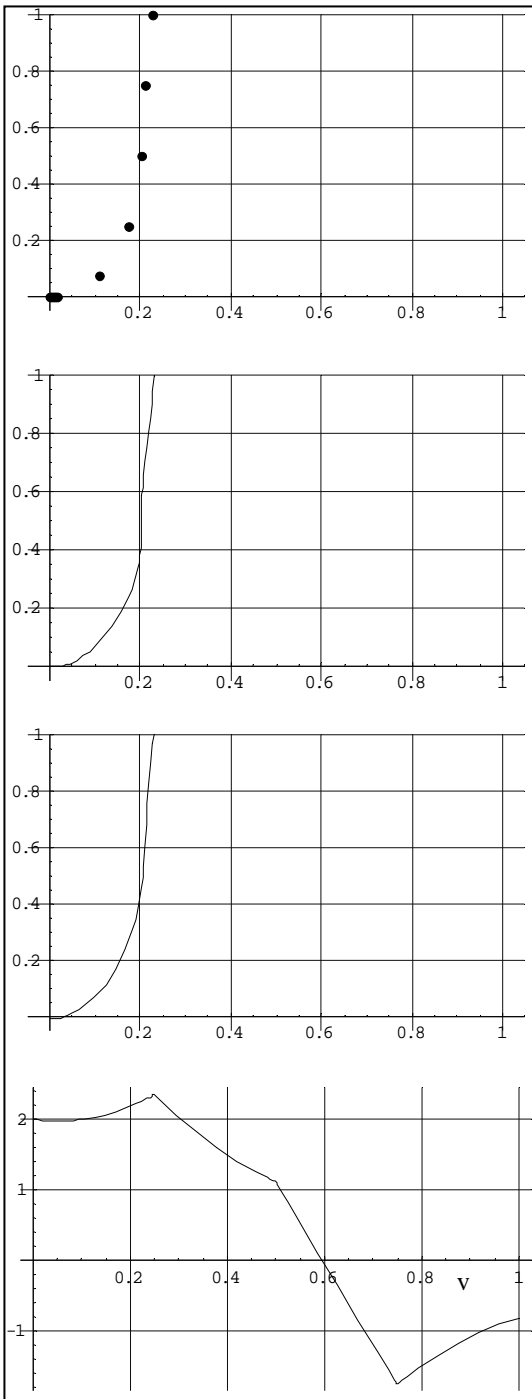


Figure 9b - Series 60 - Station 2
 Points from the offsets table (upper), bi-cubic approximation without tangent constraint on the keel (middle upper); with constraint (middle lower); curvature (lower) versus arc length

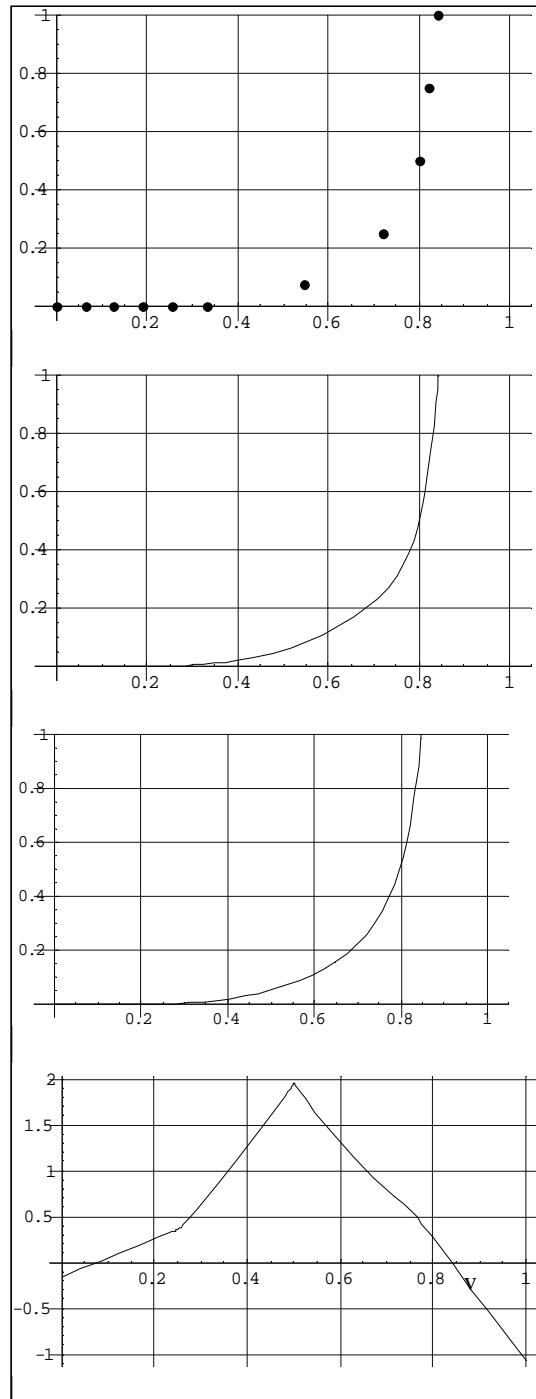


Figure 9c - Series 60 - Station 6
 Points from the offsets table (upper), bi-cubic approximation without tangent constraint on the keel (middle upper); with constraint (middle lower); curvature (lower) versus arc length

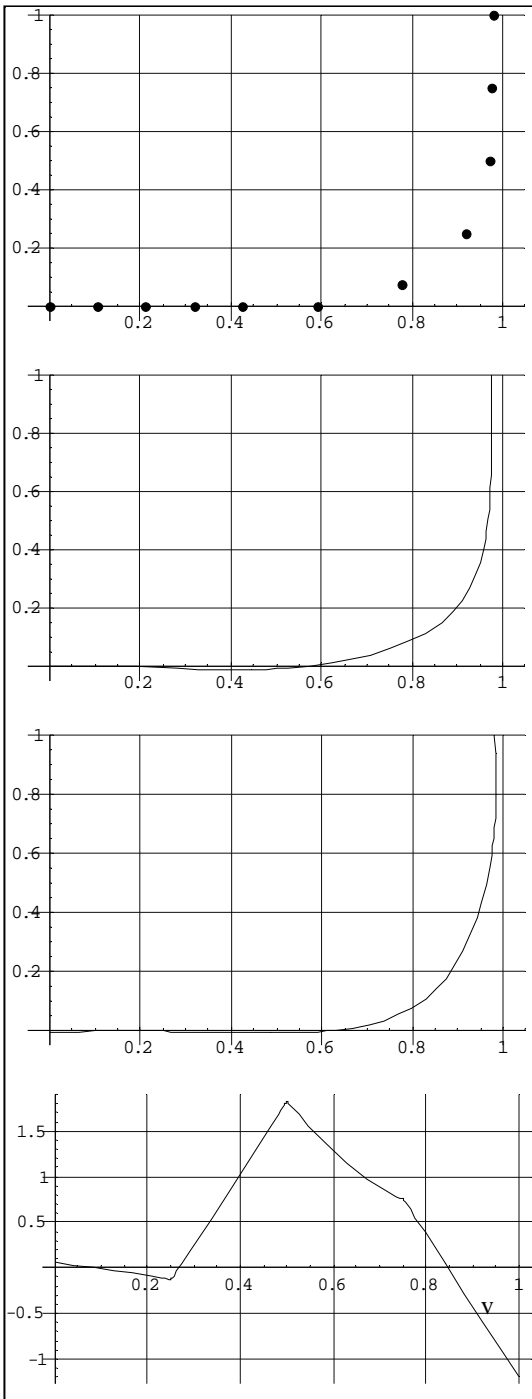


Figure 9d - Series 60 - Station 8
 Points from the offsets table (upper), bi-cubic approximation without tangent constraint on the keel (middle upper); with constraint (middle lower); curvature (lower) versus arc length

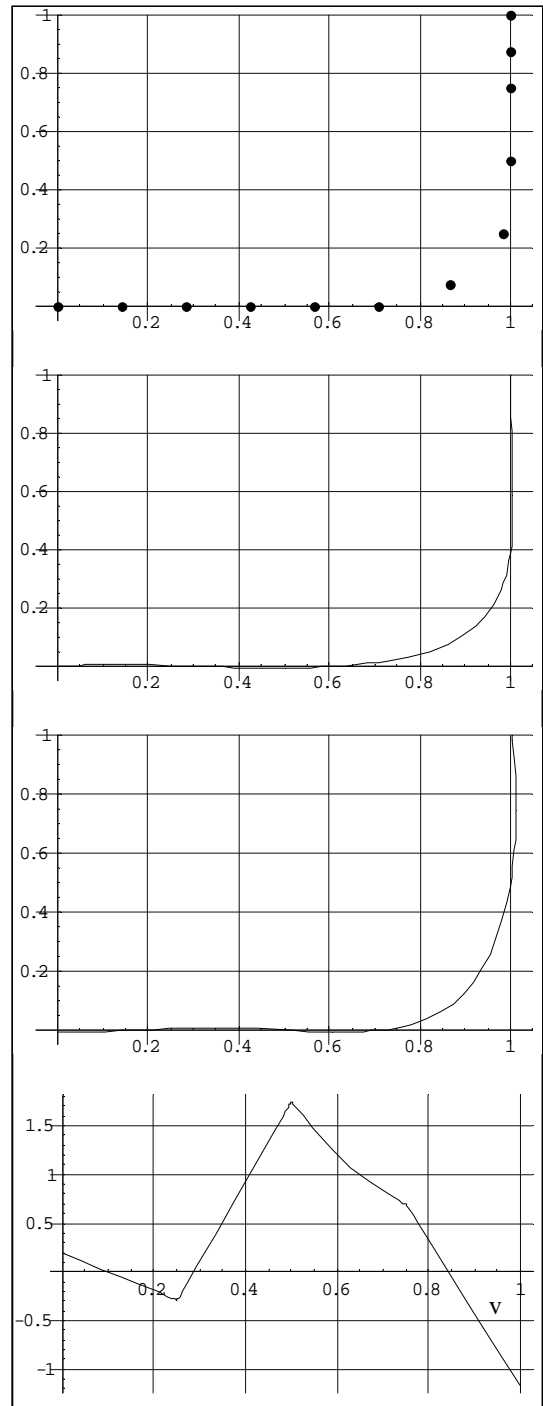


Figure 9e - Series 60 - Station 10
 Points from the offsets table (upper), bi-cubic approximation without tangent constraint on the keel (middle upper); with constraint (middle lower); curvature (lower) versus arc length

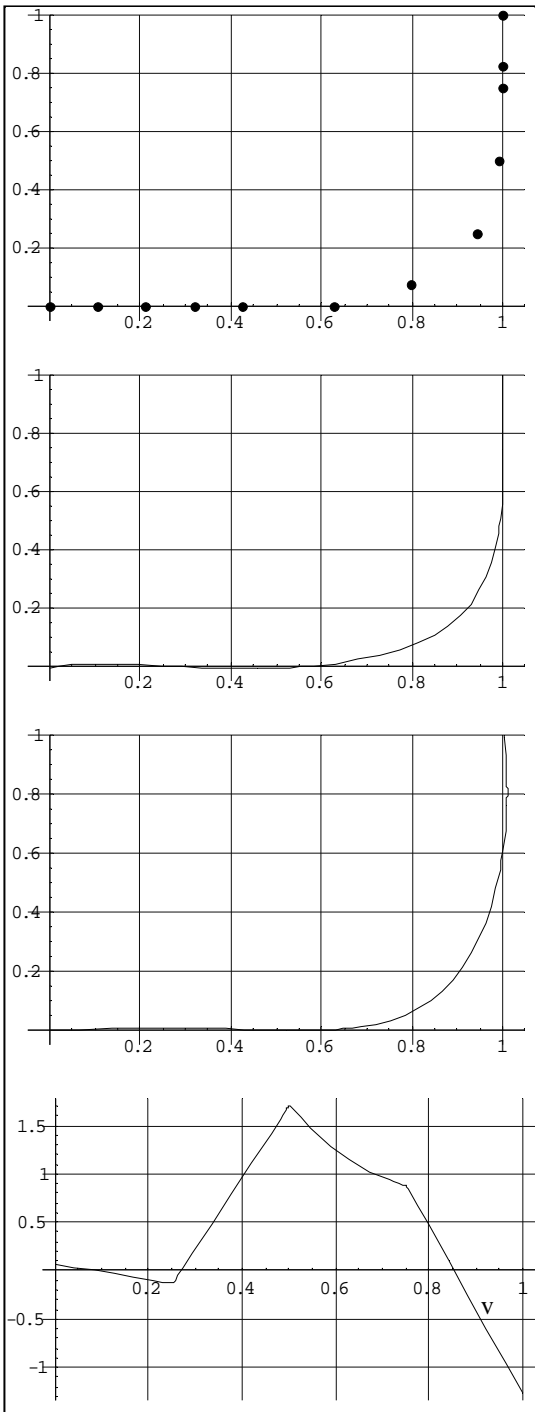


Figure 9f - Series 60 - Station 12
 Points from the offsets table (upper), bi-cubic approximation without tangent constraint on the keel (middle upper); with constraint (middle lower); curvature (lower) versus arc length

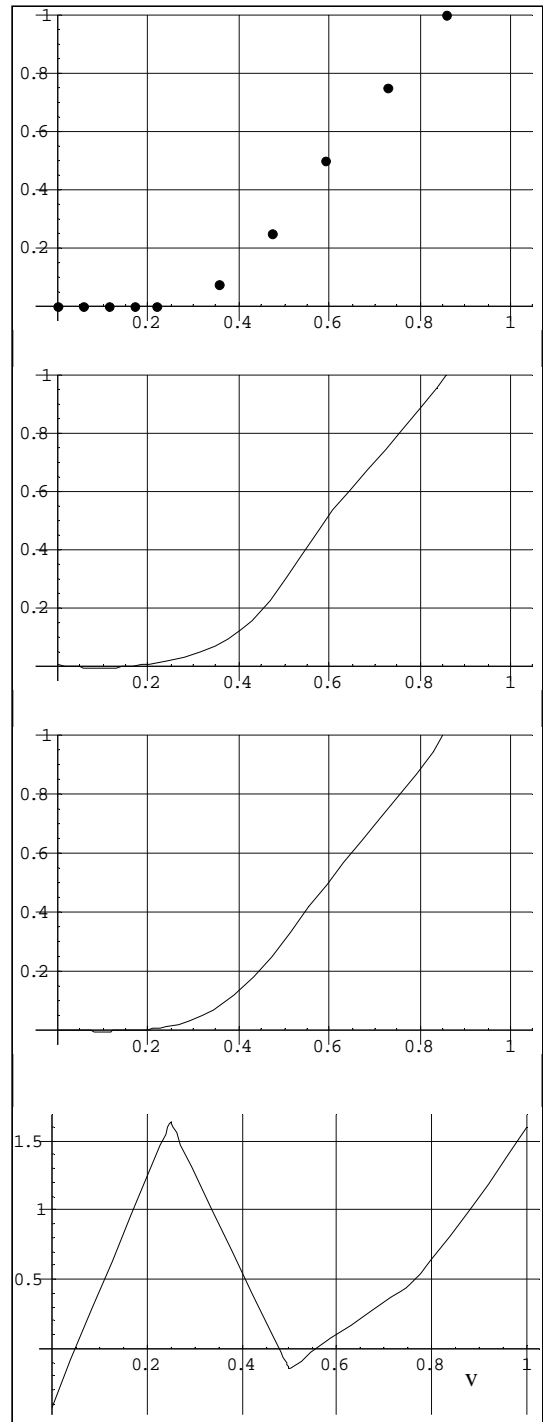


Figure 9g - Series 60 - Station 16
 Points from the offsets table (upper), bi-cubic approximation without tangent constraint on the keel (middle upper); with constraint (middle lower); curvature (lower) versus arc length

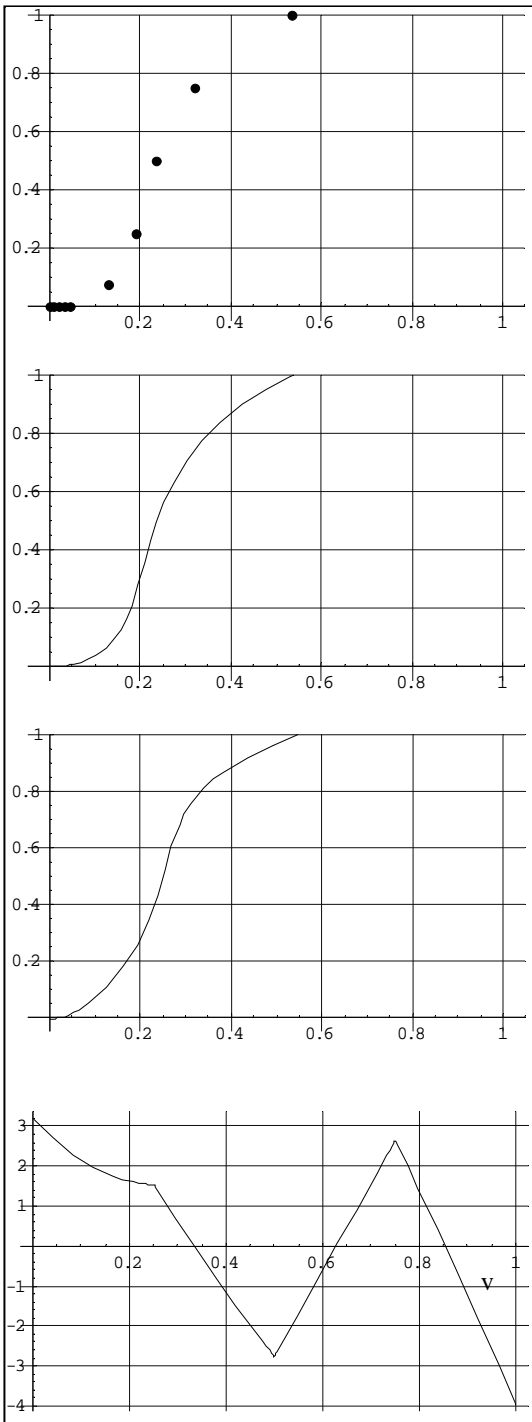


Figure 9h - Series 60 - Station 18
 Points from the offsets table (upper), bi-cubic approximation without tangent constraint on the keel (middle upper); with constraint (middle lower); curvature (lower) versus arc length

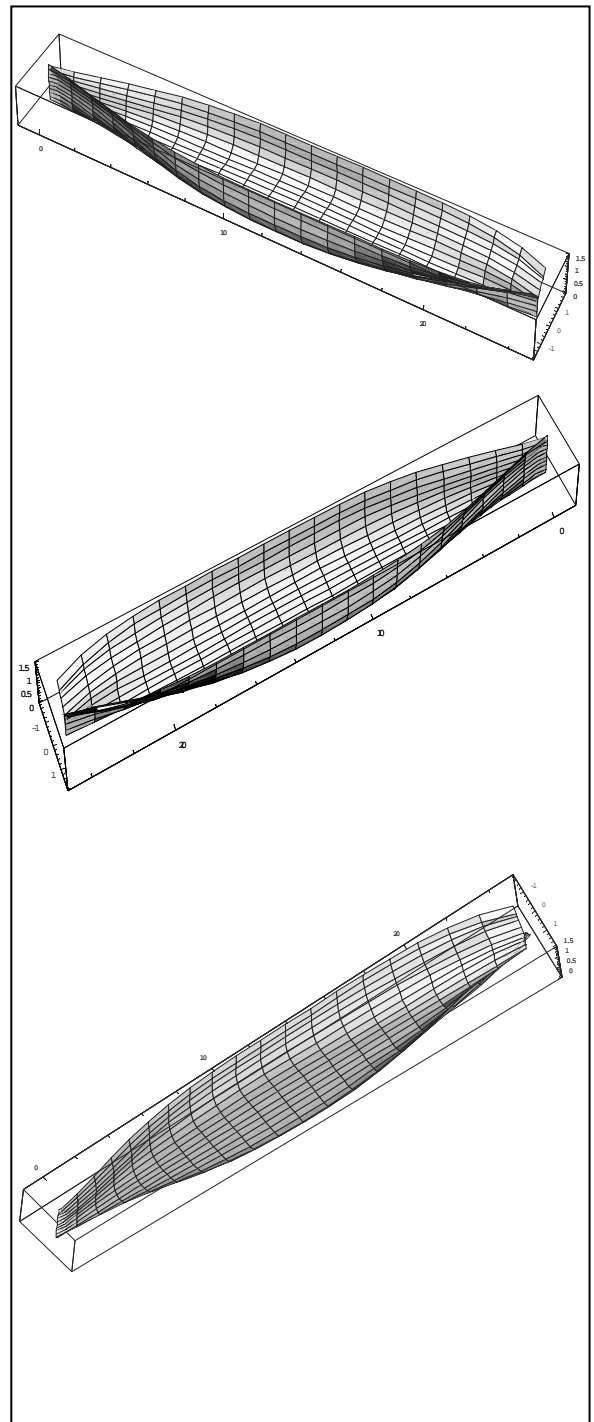


Figure 10 - Series 60 - Approximation including bow and stern - views of a grid