FUZZY PREFERENCE RELATIONS AND THEIR NAVAL ENGINEERING APPLICATIONS

Prof. Dr. R. C. Botter
Associated Professor
Naval and Ocean Engineering Department
University of São Paulo
Av. Prof. Melo Moraes 223, São Paulo, SP, Brazil,
Email: rcbotter@usp.br Phone: 55-11-3091-5340 Fax: 55-11-3091-5717

Prof. P. Ya. Ekel, D.Sc. (hab.), Ph.D.
Full Professor
Graduate Program in Electrical Engineering
Pontifical Catholic University of Minas Gerais
Ave. Dom José Gaspar, 500, 30535-610, Belo Horizonte, MG, Brazil
E-mail: ekel@pucminas.br Phone: +55-31-3319-4305 Fax: +55-31-3319-4225

In the naval engineering practice, there are many problems in which solution consequences cannot be estimated on the basis of a single criterion. As an example, it is possible to indicate the ship/cargo/route choice to invest money. These problems are of a multiattribute character. For instance, while solving the problem of ship/cargo/route, the technological, economic, social, political, logistic, etc. aspects are to be considered to make decisions justified from the different points of view. Moreover, such factors as availability, capacity, trustworthiness, flexibility, inventory, know-how, etc. are important elements of the decision making process. The majority of these factors can be reflected only by the description of qualitative character based on experience, knowledge, and intuition of involved professionals. This generates the necessity of the evaluation, comparison, choice, and/or ordering of alternatives, which correspond to the preferences of a decision maker (individual or group). Considering that a manner of human thinking and preference appreciation is vague, inexact, and subjective, the theory of fuzzy sets can serve as an important tool in the naval engineering problems. Its application to preference modeling provides a flexible structure, which allows one to deal with "fuzziness" of the appreciations and incorporate more human consistency in the applied models.

Three approaches to processing of fuzzy preferences relations are considered in the paper. The first approach is of a lexicographic character and consists in step-by-step consideration of criteria. The second technique is related to constructing and analyzing a membership function of a subset of nondominated alternatives on the basis of simultaneous considering all criteria. The third technique is based on building and aggregating membership functions of subsets of nondominated alternatives for each criterion. The consideration of different approaches is aimed at placing at decision maker's disposal the possibility to choice the most adequate approach.

The results of the paper are illustrated by considering a problem of choosing a ship/cargo/route to operate and invest a limited amount of money.

Key-words: Shipping investment, Multiattribute decision making, Fuzzy preference relations.

1. INTRODUCTION

In the naval engineering practice, there are many problems in which solution consequences cannot be estimated on the basis of a single criterion. As an example, it is possible to indicate: evaluation, comparison, choice, and/or ordering of schemes for investment ways to put money in shipping industry. These problems are of a multiattribute character. For instance, while solving the problem of ship/cargo/route, the technological, economic, social, political, logistic, etc. aspects are to be considered to make decisions justified from the different points of view. Moreover, such factors as availability, capacity, trustworthiness, flexibility, inventory, know-how, etc. are important elements of the decision making process. The majority of these factors can be reflected only by the description of qualitative character based on experience, knowledge, and intuition of involved professionals. This generates the necessity of the evaluation, comparison, choice, and/or ordering of alternatives, which correspond to the preferences of a decision maker (individual or group). Considering that a manner of human thinking and preference appreciation is vague, inexact, and subjective, the theory of fuzzy sets can serve as an important tool in solving naval engineering problems.

The application of fuzzy set theory to preference modeling [1,2] provides a flexible structure, which allows one to deal with "fuzziness" of the appreciations and incorporate more human consistency in the applied models. The utility of using fuzzy set theory is also defined by its linguistic aspect [1,2] applicable for different decision problems and different preference structures. Taking this into account, three approaches to processing of fuzzy preference relations (with direct or indirect their ordering) are considered in the present paper. The first approach is of a lexicographic character and consists in step-by-step consideration of criteria. The second technique is related to constructing and analyzing a membership function of a subset of nondominated alternatives on the basis of simultaneous considering all criteria. The third technique is based on building and aggregating membership functions of subsets of nondominated alternatives for each criterion. The consideration of different approaches is aimed at placing at decision maker's disposal the possibility to choice the most adequate approach.

The use of the results of this work will allow one to increase the adequacy of applied models and, as a result, their credibility and the factual efficiency of decisions based on their analysis. These results are illustrated by considering a problem of choosing a way for decide the best investment in shipping industry.

2. PROBLEM FORMULATION

Assume we are given a set X of alternatives, which are to be examined by q criteria of quantitative and/or qualitative nature to make a choice among alternatives. The problem of decision-making is presented by a pair $X_k, X_l \in X$ where $R = \{R_1, ..., R_q\}$ is a vector fuzzy preference relation [3,4]. In this case, we have

$$R_p = [X \times X, \mu_{R_p}(X_k, X_l)], \quad p = 1,...,q, \quad X_k, X_l \in X \ , \tag{1}$$

where $\mu_{R_p}(X_k, X_l)$ is a membership function of fuzzy preference relation.

In (1), R_p is defined as a fuzzy set of all pairs of the Cartesian product $X \times X$, such that the membership function $\mu_{R_p}(X_k, X_l)$ represents the degree to which X_k weakly dominates X_l, i.e., the degree to which X_k is at least as good as X_l (X_k is not worse than X_l) for the pth criterion. In a somewhat loose sense [3,5], $\mu_{R_p}(X_k, X_l)$ also represents the degree of truth of the statement "X_k is preferred over X_l".

Information given in the form (1) permits one to narrow a set of rational alternatives including in it only alternatives, which are not dominated by any alternative of X.

It is supposed in [3,4] that the matrices R_p, $p = 1,...,q$ are directly given as expert's estimates (from the interval [0, 1]) denoting the degree of preference of one alternative over the other. However, there is another, more convincing and natural, approach to obtaining these matrices. In particular, the availability of fuzzy or linguistic estimates of alternatives $\widetilde{F}_p(X_k)$, $p = 1,...,q$, $X_k \in X$ (constructed on the basis of expert estimation or on the basis of aggregating information arriving from different sources of both formal and informal character [6,7]) with the membership functions $\mu[f_p(X_k)]$, $p = 1,...,q$, $X_k \in X$ permits one to construct the matrices R_p, $p = 1,...,q$ as follows [8,9]:

$$\mu_{R_p}(X_k, X_l) = \sup_{\substack{X_k, X_l \in X \\ f_p(X_k) \geq f_p(X_l)}} \min\{\mu[f_p(X_k)], \mu[f_p(X_l)]\}, \qquad (2)$$

$$\mu_{R_p}(X_l, X_k) = \sup_{\substack{X_k, X_l \in X \\ f_p(X_l) \geq f_p(X_k)}} \min\{\mu[f_p(X_k)], \mu[f_p(X_l)]\} \qquad (3)$$

if the pth criterion is associated with maximization. If the pth criterion is associated with minimization, then (2) and (3) are written for $f_p(X_k) \leq f_p(X_l)$ and $f_p(X_l) \leq f_p(X_k)$, respectively.

3. FUZZY PREFERENCE PROCESSING

Let us consider a situation of setting up a single preference relation R. It can be represented by a strict preference relation R^S and indifferent relation R^I [3,4]. We can say that "X_k is strictly better than X_l" if $(X_k, X_l) \in R$ and $(X_l, X_k) \notin R$. The set of all these pairs is the strict preference relation R^S, and it is possible to use the inverse relation R^{-1} to obtain $R^S = R \setminus R^{-1}$.

If $(X_k, X_l) \in R^S$, then X_k dominates X_l, i.e., $X_k \succ X_l$. The alternative $X_k \in X$ is nondominated in $< X, R >$ if $(X_k, X_l) \in R^S$ for any $X_l \in X$.

If we have $\mu_R(X_k, X_l)$ as a nonstrict fuzzy preference relation, then its value is the degree of preference $X_k \succcurlyeq X_l$ for any $X_k, X_l \in X$. The membership function, which corresponds to R^S (considering that $\mu_{R^{-1}}(X_k, X_l) = \mu_R(X_l, X_k)$) is the following:

$$\mu_R^s(X_k, X_l) = \max\{\mu_R(X_k, X_l) - \mu_R(X_l, X_k), 0\}. \quad (4)$$

The expression (4) serves as the basis for the choice procedure introduced by Orlovsky [4,10]. Many authors have studied this procedure. For instance, it was shown in [11] that the Orlovsky choice procedure possesses many interesting desirable properties. Its axiomatic is given in [12,13].

The use of (4) permits one to carry out the choice of alternatives. In particular, $\mu_R^S(X_l, X_k)$ is the membership function of the fuzzy set of all X_k, which are strictly dominated by X_l. Therefore, its complement by $1-\mu_R^S(X_l, X_k)$ gives the fuzzy set of alternatives, which are not dominated by other alternatives from X. To choice the set of all alternatives, which are not dominated by other alternatives from X, it is necessary to find the intersection of all $1-\mu_R^s(X_l, X_k), X_k \in X$ on all $X_l \in X$ [4,10]. This intersection is the fuzzy set of nondominated alternatives and has a membership function

$$\mu_R^{ND}(X_k) = \inf_{X_l \in X}[1-\mu_R^S(X_l, X_k)] = 1 - \sup_{X_l \in X}\mu_R^S(X_l, X_k). \quad (5)$$

Because $\mu_R^{ND}(X_k)$ is the degree of nondominance, it is natural to obtain alternatives providing

$$X^{ND} = \{X_k^{ND} \mid X_k^{ND} \in X, \mu_R^{ND}(X_k^{ND}) = \sup_{X_k \in X}\mu_R^{ND}(X_k)\}. \quad (6)$$

Orlovsky [10] also introduced the set of nonfuzzy nondominated alternatives. In particular, if $\sup_{X_k \in X}\mu_R^{ND}(X_k) = 1$, then alternatives

$$X^{NFND} = \{X_k^{NFND} \mid X_k^{NFND} \in X, \mu_R^{ND}(X_k^{NFND}) = 1\} \quad (7)$$

are nonfuzzy nondominated and can be considered as the nonfuzzy solution of the fuzzy problem.

If the fuzzy preference relation *R* is transitive, then $X^{NFND} \neq \varnothing$. Taking this into account, it should be noted that when $\widetilde{F}_p(X_k)$ is quantitatively expressed, $X^{NFND} \neq \varnothing$. With qualitative $\widetilde{F}_p(X_k)$ it is possible to have $X^{NFND} = \varnothing$ under intransitivity of *R* [14]. It permits one to detect contradictions in expert's estimates.

When R is a vector fuzzy preference relation, the expressions (4)-(6) can be applied if we take $R = \bigcap_{p=1}^{q} R_p$, i.e.,

$$\mu_R(X_k, X_l) = \min_{1 \le p \le q}\mu_{R_p}(X_k, X_l), \quad X_k, X_l \in X. \quad (8)$$

When using this intersection, the set X^{ND} fulfils the role of a Pareto set [4]. Its contraction is possible on the basis of differentiating the importance of R_p, $p=1,...,q$ with the use of the following convolution (aggregation of monobjective preference relations) [4]:

$$\mu_T(X_k, X_l) = \sum_{p=1}^{q} \lambda_p \mu_{R_p}(X_k, X_l), \quad X_k, X_l \in X, \tag{9}$$

where $\lambda_p \geq 0$, $p=1,...,q$ are weights (importance factors) for the corresponding criteria normalized as

$$\sum_{p=1}^{q} \lambda_p = 1. \tag{10}$$

The construction of $\mu_T(X_k, X_l)$, $X_k, X_l \in X$ allows one to obtain the membership function $\mu_T^{ND}(X_k)$ of the set of nondominated alternatives according to an expression similar to (4). The intersection of $\mu_R^{ND}(X_k)$ and $\mu_T^{ND}(X_k)$ defined as

$$\mu^{ND}(X_k) = \min\{\mu_R^{ND}(X_k), \mu_T^{ND}(X_k)\}, \quad X_k \in X \tag{11}$$

provides us with

$$X^{ND} = \{X_k^{ND} \mid X_k^{ND} \in X, \mu^{ND}(X_k^{ND}) = \sup_{X_k \in X} \mu^{ND}(X_k)\}. \tag{12}$$

The expressions (5) and (6) can serve as the basis for building another procedure, which is of a lexicographic character [15,16]. This procedure is associated with step-by-step introduction of criteria for comparing alternatives. The procedure permits one to construct a sequence $X^1, X^2, ..., X^q$ so that $X \supseteq X^1 \supseteq X^2 \supseteq ... \supseteq X^q$ with the use of the following expressions:

$$\mu_{R_p}^{ND}(X_k) = \inf_{X_l \in X^{p-1}} [1 - \mu_{R_p}^{S}(X_l, X_k)] = 1 - \sup_{X_l \in X^{p-1}} \mu_{R_p}^{S}(X_l, X_k), \quad p=1,...,q, \tag{13}$$

$$X^p = \{X_k^{ND,p} \mid X_k^{ND,p} \in X^{p-1}, \mu_{R_p}^{ND}(X_k^{ND,p}) = \sup_{X_l \in X^{p-1}} \mu_{R_p}^{ND}(X_k)\}. \tag{14}$$

It should be noted that if R_p is transitive, we can bypass the pairwise comparison of alternatives at the *p*th step. In this situation, the comparison can be done on a serial basis (the direct use of (2) and (3)) with memorizing the best alternatives.

The described choice procedures have found practical applications [17]. However, it is possible to propose the third procedure to contract the decision uncertainty region.

The use of (5) represented in the form

$$\mu_{R_p}^{ND}(X_k) = 1 - \sup_{X_l \in X} \mu_{R_p}^{S}(X_l, X_k), \quad p=1,...,q \tag{15}$$

permits one to construct the membership functions of the set of nondominated alternatives for each fuzzy preference relation.

The membership functions $\mu_{R_p}^{ND}(X_k)$, $p=1,...,q$ play a role identical to membership functions replacing objective functions $F_p(X)$, $p=1,...,q$ in solving traditional problems of multiobjective optimization [15,18] on the basis of the Bellman-Zadeh approach to decision making in a fuzzy environment [1,2]. Therefore, it is possible to construct

$$\mu^{ND}(X_k)=\min_{1\le p\le q}\mu_{R_p}^{ND}(X_k) \tag{16}$$

to obtain X^{ND}.

If necessary to differentiate the importance of different preference relations, it is possible to transform (16) as

$$\mu^{ND}(X_k)=\min_{1\le p\le q}[\mu_{R_p}^{ND}(X_k)]^{\lambda_p}. \tag{17}$$

The utilization of (17) does not require the normalization of λ_p, $p=1,...,q$ in the way similar to (10).

The use of the second procedure may lead to solutions different from results obtained on the basis of the first procedure. However, solutions based on the first and third procedures, which have a single generic basis, may at time also be different. At the same time, the third procedure is more preferential from the substantial point of view. In particular, the use of the first procedure can lead to choosing alternatives with the degree of nondominance equal to one, though these alternatives are not the best ones from the point of view of all preference relations. The third procedure can give this result only for alternatives that are the best solutions from the point of view of all fuzzy preference relations. This is illustrated by an example given below.

It should be stressed that the fact of the possibility to obtain different solutions on the basis of different approaches is to be considered natural, and the choice of the approach is a prerogative of a decision maker.

4. ILLUSTRATIVE EXAMPLE

The shipping industry has been atracted a large sum of investments year by year, ordering new vessels to keep a good level of service in the sea transportation.

We can find some good and profits sea transportation trades to be analysed, such as container, grain, petroleum and fuel, chemicals, ore and coal transportation.

In 2004 the world could see a high demmand for grain and ore vessels, which resulted in a very high increase of the transportation tariffs per ton.

For this reason the investors have begun to order vessels of different types and sizes to supply the marktes around the world. The decision to select a ship and its trade is not easy do. It can be based in some criterias such as minimum capital employed to build a ship, maximum freigth in shipping route trades, maximum realibility of shiping markets considered, maximum cargoes demand forecast and minimum number of shipping companies competitors.

To give a technical support the decision of which ship to buy, the results of the paper are illustrated by considering a problem of choosing a ship/cargo/route to operate and invest a limited amount of money in one of three selected trades:

1. Coal Transportation ;
2. Ore Transportation;
3. Pulp Transportation.

The decision is to be made on the basis of the following criteria:

1. Acquisition cost;
2. Cargo protection level;
3. Competition level.

The membership functions corresponding to the fuzzy values (*very small* – VS, *small* – S, *moderate* – M, *large* – L, and *very large* –VL) of the linguistic variables "*Financial Expenditures*", "*Job Execution Quickness*", and "*Job Execution Quality*", which can be used to estimate $\widetilde{F}_p(X_k)$, $p = 1, 2, 3$, $k = 1, 2, 3$, are given in Figure 1 with $a = 0.95$, $b = 0.90$, and $c = 0.60$.

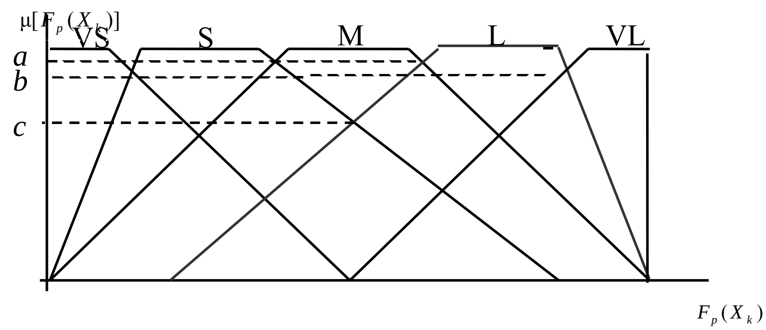

Figure 1.: Membership functions of fuzzy values of linguistic variables

Assume that the alternatives have received the following estimates: $\widetilde{F}_1(X_1) = \mathrm{M}$, $\widetilde{F}_1(X_2) = \mathrm{S}$, $\widetilde{F}_1(X_3) = \mathrm{L}$, $\widetilde{F}_2(X_1) = \mathrm{S}$, $\widetilde{F}_2(X_2) = \mathrm{M}$, $\widetilde{F}_2(X_3) = \mathrm{L}$, $\widetilde{F}_3(X_1) = \mathrm{M}$, $\widetilde{F}_3(X_2) = \mathrm{M}$, and $\widetilde{F}_3(X_3) = \mathrm{S}$. Then, using (2) and (3) with taking into account the need to minimize $\widetilde{F}_1(X_k)$ and $\widetilde{F}_3(X_k)$ and to maximize $\widetilde{F}_2(X_k)$, we can construct the fuzzy preference relation R_1

$$\mu_{R_1}(X_k, X_l) = \begin{bmatrix} 1 & 0.95 & 1 \\ 1 & 1 & 1 \\ 0.95 & 0.60 & 1 \end{bmatrix}, \qquad (18)$$

the fuzzy preference relation R_2

$$\mu_{R_2}(X_k, X_l) = \begin{bmatrix} 1 & 0.95 & 0.60 \\ 1 & 1 & 0.95 \\ 1 & 1 & 1 \end{bmatrix}, \tag{19}$$

and the fuzzy preference relation R_3

$$\mu_{R_3}(X_k, X_l) = \begin{bmatrix} 1 & 1 & 0.95 \\ 1 & 1 & 0.95 \\ 1 & 1 & 1 \end{bmatrix}. \tag{20}$$

A result of intersection of (18)-(20) is

$$\mu_R(X_k, X_l) = \begin{bmatrix} 1 & 0.95 & 0.60 \\ 1 & 1 & 0.95 \\ 0.95 & 0.60 & 1 \end{bmatrix}, \tag{21}$$

which permits us, using (4), to construct the strict preference relation with:

$$\mu_R^S(X_k, X_l) = \begin{bmatrix} 0 & 0 & 0 \\ 0.05 & 0 & 0.35 \\ 0.35 & 0 & 0 \end{bmatrix}. \tag{22}$$

Applying (5) to (22), it is possible to obtain the membership function of the set of nondominated alternatives

$$\mu_R^{ND}(X_k, X_l) = [0.65 \quad 1 \quad 0.65], \tag{23}$$

which leads to $X^{ND} = \{X_2\}$.

Let us consider the second approach if the criteria are arranged, for example, in the following order of their importance: $p = 1$, $p = 2$, and $p = 3$.

Using (18), it is possible to form the strict fuzzy preference relation with

$$\mu_{R_1}^S(X_k, X_l) = \begin{bmatrix} 0 & 0 & 0.05 \\ 0.05 & 0 & 0.40 \\ 0 & 0 & 0 \end{bmatrix} \tag{24}$$

Following (13) and (14), we obtain on the basis of (24)

$$\mu_{R_1}^{ND}(X_k) = [0.95 \quad 1 \quad 0.60] \tag{25}$$

and $X^1 = \{X_2\}$.

Finally, let us consider the application of the third approach. The membership function of the subset of nondominated alternatives for the first fuzzy preference relation $\mu_{R_1}^{ND}(X_k)$ is (25).

Using (19), it is possible to form the strict fuzzy preference relation with

$$\mu^{S}_{R_2}(X_k, X_l) = \begin{bmatrix} 0 & 0 & 0 \\ 0.05 & 0 & 0 \\ 0.40 & 0.05 & 0 \end{bmatrix} \quad (26)$$

to obtain

$$\mu^{ND}_{R_2}(X_k) = [0.60 \quad 0.95 \quad 1]. \quad (27)$$

In an analogous way, the use of (20) leads to

$$\mu^{ND}_{R_3}(X_k) = [1 \quad 0.95 \quad 0.95]. \quad (28)$$

The intersection of (25), (27), and (28) permits us to construct

$$\mu^{ND}_{R_3}(X_k) = [0.60 \quad 0.95 \quad 0.60] \quad (32)$$

to get $X^{ND} = \{X_2\}$ as well.

5. CONCLUSION

Since naval engineering problems are of a multiattribute character and include factors with the description of qualitative character based on experience, knowledge, and intuition of involved professionals, it is expeditious to apply fuzzy preference modeling to the analysis of these problems. Taking this into account, three approaches to processing of fuzzy preferences relations have been considered in the paper. The first approach is associated with analyzing a membership function of a subset of nondominated alternatives with simultaneous considering all criteria. The second technique is of a lexicographic character and consists in step-by-step introducing criteria. The third technique is based on aggregating membership functions of subsets of nondominated alternatives for each criterion. It is natural that the choice of the approach is a prerogative of a decision maker. The results of the paper have been illustrated by considering a problem of ship/cargo/route to operate and invest a limited amount of money.

4. REFERENCES

[1] H.J. Zimmermann, **Fuzzy Set Theory and Its Application**, Kluwer Academic Publishers, 1990.

[2] W. Pedrycz and F. Gomide, **An Introduction to Fuzzy Sets: Analysis and Design**, MIT Press, 1998.

[3] J. Fodor and M. Roubens, **Fuzzy Preference Modelling and Multicriteria Decision Support**, Kluwer Academic Publishers, 1994.

[4] S.A. Orlovsky, **Problems of Decision Making with Fuzzy Information** (in Russian), Nauka, 1981.

[5] P. Kulshreshtha and B. Shekar, Interrelations among fuzzy preference-based choice functions and significance of rationality conditions: A taxonomic and intuitive perspective, **Fuzzy Sets and Systems**, vol. 109, 2000, pp. 429-445.

[6] P.Ya. Ekel and V.A. Popov, Consideration of the uncertainty factor in problems of modelling and optimizing electrical networks, **Power Engineering**, vol. 23, 1985, pp. 45-52.

[7] P.Ya. Ekel, Taking into account the uncertainty factor in problems of modelling and optimizing complex systems, **Advances in Modelling and Analysis**, vol. C-43, 1994, pp. 11-22.

[8] P. Ekel, W. Pedrycz, and R. Schinzinger, A general approach to solving a wide class of fuzzy optimization problems, **Fuzzy Sets and Systems**, vol. 97, 1998, pp. 49-66.

[9] P.Ya. Ekel, Methods of decision making in fuzzy environment and their applications, **Nonlinear Analysis**, vol. 47, 2001, pp. 979-990.

[10] S.A. Orlovsky, Decision making with a fuzzy preference relation, **Fuzzy Sets and Systems**, vol. 3, 1978, pp. 155-167.

[11] C.R. Barrett, P.K. Pattanaik, and M. Salles, On choosing rationally when preferences are fuzzy, **Fuzzy Sets and Systems**, vol. 34, 1990, pp. 197-212.

[12] D. Bouyssou, Acyclic fuzzy preference and the Orlovsky choice function: A note, **Fuzzy Sets and Systems**, vol. 89, 1997, pp. 107-111.

[13] K. Sengupta, Fuzzy preference and Orlovsky choice procedure, **Fuzzy Sets and Systems**, vol. 93, 1998, pp. 231-234.

[14] Z. Switalski, Transitivity of fuzzy preference relations – an empirical study, **Fuzzy Sets and Systems**, vol. 118, 2001, pp. 503-508.

[15] P.Ya. Ekel, Fuzzy sets and models of decision making, **International Journal of Computers and Mathematics with Applications**, vol. 44, 2002, pp. 863-875.

[16] P. Ekel, E. Galperin, R. Palhares, C. Campos, and M. Silva, Fuzzy coefficients and fuzzy preference relations in models of decision making, in **Lecture Notes in Artificial Intelligence**, Springer Werlag, vol. 2773, 2003, pp. 229-236.

[17] P.Ya. Ekel and L.D.B. Terra, Fuzzy preference relations: methods and power engineering applications, **OPSEARCH**, vol. 39, 2002, pp. 34-45.

[18] P.Ya. Ekel and E.A. Galperin, Box-triangular multiobjective linear programs for resource allocation with application to load management and energy market problems, **Mathematical and Computer Modelling**, vol. 37, 2003, pp. 1-17.