Determination of Floating Units Hydrostatic Properties

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Abstract

Due to a constant need of vessel constructions, whether by market demand or a fleet renewal, the computational programs development which can facilitate and even help a ship project, become an interesting and useful tool, aiding in several ship project phases.

To demonstrate the shipbuilding growth, on January 1st, 2003 the world orderbook was about 2248 ships, going up to 3338 on July 1st, 2004 [2]. 2004 was an excellent year for shipbuilding market, increasing most of orderbooks for larger vessels. According to Clarkson Research Studies [1], it is noted that bulkcarriers, tankers and container ships fleet have been grown for last two years, achieving incredible levels as never seen before.

In order to contribute with the market growth, specially the oil market one, it was generated a computational program called NAVSTAB which is able to calculate floating units hydrostatic properties. These properties are obtained after NAVSTAB generates panels for a hull representation based on vessel's cotes provided previously by user. With these panels, it is possible to make a vectorial calculus in order to represent the hydrostatic curves.

Although the developed program has been created at first to analyze ships only, after some changes in its structure, platform properties can also be determined.

Some comparisons and validations were made with some cotes and hydrostatic curves known from literature for two tankers and the results achieved by NAVSTAB were extremely close to them, showing its capabilities and accuracy. In the platform comparison, NAVSTAB executes a vectorial calculation based on panels provided, once only for these floating units panels shall be obtained externally.

In this same program it was also implemented a method to obtain vessel properties when it moves or rotates in relation to axes x, y and z as follows: surge, sway, heave, roll, pitch e yaw.

The program was developed at Department of Naval Architecture and Ocean Engineering of University of São Paulo, Brazil.

1 – Introduction

The hydrostatic properties of a vessel are extremely important within its project. With those, ship behavior in relation to its static stability, for instance, can be analyzed.

With an increasing necessity on the vessel's market demand, it was developed a way to determinate these ship properties based on its own cotes. The program NAVSTAB main purpose is to determine the hydrostatic properties for tankers, once in Brazil they are largely used due to a constant growth of oil exploration in deep water.

With the ship cotes, using an interpolation process, NAVSTAB generates a panel mesh that later, with some vectorial calculus properties, gives, as a result, the ship hydrostatic curves for many different drafts defined by the user, including the projected one.

Thus, joining an interpolation method with panel vectorial calculus, it is possible to determine:

- Underwater (immersed) volume;
- LCF – Longitudinal center of flotation;

- TCF – Transversal center of flotation;
- LCB – Longitudinal center of buoyancy;
- TCB – Transversal center of buoyancy;
- KB – High of center of buoyancy (in relation to keel);
- Longitudinal and transversal inertial moments of the buoyancy plan;
- Floating surface area;
- Wet surface;
- Longitudinal and transversal BM (metacentric radius).

The interpolation calculus method as well as properties mentioned above will be presented during this paper.

2 – Panels Interpolation and Generation

With ship cotes, it is possible to delineate its stations and water line contours. The ship can be divided in many stations and water lines that the user wants to. For this, it is just needed to apply an interpolation concept.

In this topic, a *spline* method will be presented to explain how interpolation is used by NAVSTAB and also how it is applied to different floating units geometry.

It is important to call attention that the interpolation method is only utilized for the ship geometry delineation and not for calculating the hydrostatic properties. In platform cases, the panels are determinate externally and then specified to the program. The interpolation process developed does not work for platforms.

2.1 – *Spline* Functions for Interpolation

According to [3], a *spline* function $S_p(x)$ of p degrees with nodes in points x_i (i = 0, 1,..., n) is defined by the following conditions:

- In each sub-interval of points $[x_i, x_{i+1}]$, where i = 0, 1,..., (n-1), $S_p(x)$ is a polynomial of p degrees: $s_p(x)$;
- $S_p(x)$ is continuous and has a continuous derivate until (p-1) order within the considered interval;
- $S_p(x)$ passes through the points in the interval given.

There are some *spline* functions which are appropriated for interpolation. They are: 1st degree (linear function), 2nd degree (quadratic function) and 3rd degree (cubic function).

When considering a linear *spline* to interpolate some points, it is easy to identify as a big drawback a discontinuous first derivate at nodes. The quadratic function has continuous derivates until first order, but it does not assure that second order derivates is continuous too. Therefore, the most appropriated *spline* and also applied in the NAVSTAB calculation is a *spline* of third degree or a cubic *spline* interpolation function.

The third-degree *spline* has two continuous derivates, not allowing peaks or abrupt changes at nodes curvature in the interpolation function.

For each interval between any two points $[x_k, x_{k+1}]$ with k = 0, 1, ..., (n-1), with n been the number of points given where the *spline* function must pass through, there is a polynomial $s_k(x)$ of third degree, written as:

$$s_k(x) = a_k (x - x_k)^3 + b_k (x - x_k)^2 + c_k (x - x_k) + d_k \tag{1}$$

where a_k, b_k, c_k and d_k are coefficients that might be found out for each k = 0 to n-1 value. Using:

$$g_k = s''_k\left(x_k\right) \quad \text{and} \quad h_k = x_k - x_{k-1} \tag{2}$$

it is possible to figure out a_k, b_k, c_k e d_k values, by the following expressions:

$$a_k = \frac{g_k - g_{k-1}}{6h_k},\ b_k = \frac{g_k}{2},\ c_k = \left[\frac{y_k - y_{k-1}}{h_k} + \frac{2h_k g_k + g_{k-1} h_k}{6}\right],\ d_k = y_k \tag{3}$$

and g_k value can be determinate with the system Ax = b solution, where:

$$A = \left(\begin{array}{c:c:c:c:c}
h_1 & 2\left(h_1 + h_2\right) & h_2 & 0 & 0 \\ \hdashline
0 & h_2 & 2\left(h_2 + h_3\right) & h_3 & 0 \\ \hdashline
0 & 0 & \ddots & \ddots & 0 \\ \hdashline
\vdots & \vdots & \ddots & \ddots & \vdots \\ \hdashline
0 & 0 & h_{n-1} & 2\left(h_{n-1} + h_n\right) & h_n
\end{array}\right) \tag{4}$$

$$b = \begin{pmatrix}
\dfrac{y_2 - y_1}{h_2} - \dfrac{y_1 - y_0}{h_1} \\
\dfrac{y_3 - y_2}{h_3} - \dfrac{y_2 - y_1}{h_2} \\
\vdots \\
\dfrac{y_n - y_{n-1}}{h_n} - \dfrac{y_{n-1} - y_{n-2}}{h_{n-1}}
\end{pmatrix} \tag{5}$$

and:

$$x = \left(g_0, g_1, \ldots, g_n\right)^T \tag{6}$$

With this system solution, we can easily obtain a cubic *spline* function which interpolates panels.

2.2 – Panels Generation

On knowing that it is possible to interpolate points with functions, it will be described now how it was implemented within NAVSTAB in order to generate panels. First of all, some points distributed in the same water line must be provided by user, as exemplified in Fig. 1.

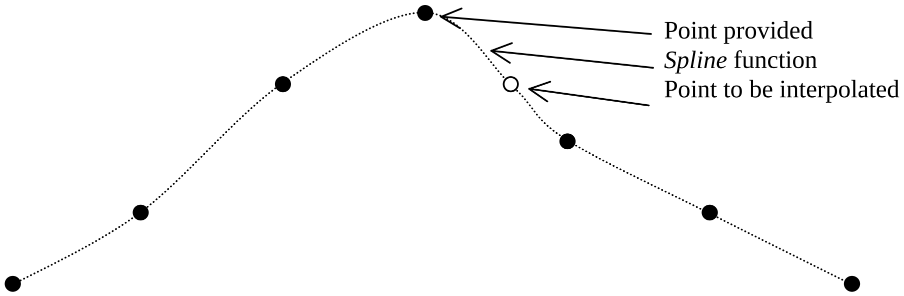


Fig. 1: Example of an interpolated *spline* function (water line contours).

Based on points provided, the coordinates of any other intermediate point can be defined inside any interval by generating *splines* functions. This allows, in the case of intermediate stations and for the same existing water line or for another one generated by the program, calculate intersection points distances in relation to a longitudinal axis that goes from stern to bow.

Consider now that vessel cotes have been provided to NAVSTAB, containing ten water lines, ten stations and its correspondent distances in relation to the longitudinal axis.

If it is necessary to better describe the considered vessel, NAVSTAB can interpolate, for each water line generated, as many stations as user has specified. Hence, with a better described ship, the determinate hydrostatics properties will be much more accurate. This is the main reason why panels are created. They are situated between stations and water lines. In the program's case, panels have always four points and are generated with a normal vector perpendicular to its surface pointing out of the hull.

Let us consider now two water lines and two stations provided and, between them, one water line and one station, both generated by NAVSTAB, as represented in Fig. 2.

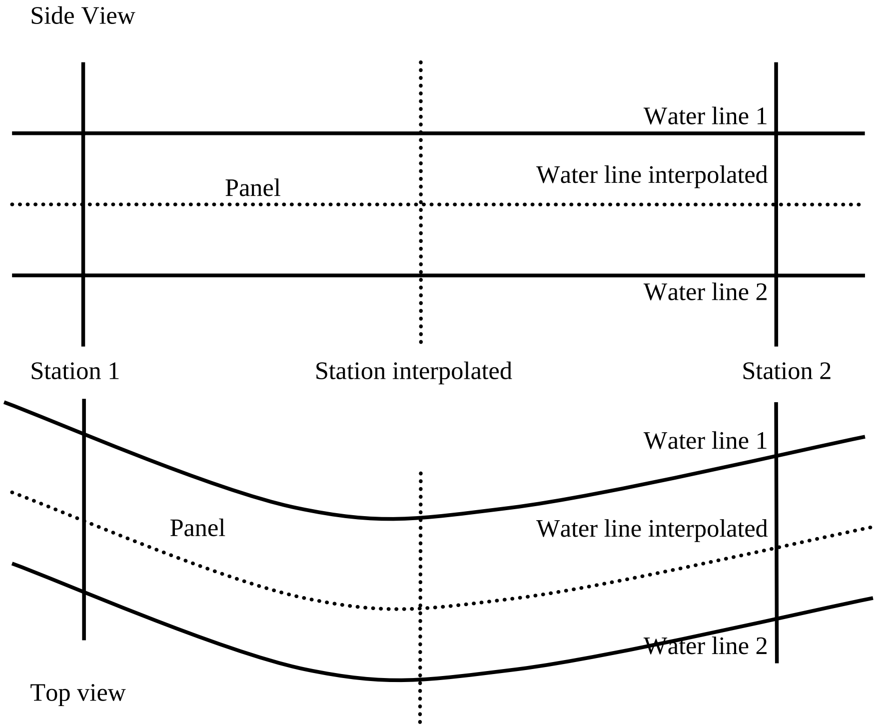


Fig. 2: Generated panel by NAVSTAB with both station and water line interpolated.

Generated panels are exactly between interpolated water lines and stations. Thus, it is easy to confirm that more water lines and more stations created (smaller distances between them), the smaller panels are and, therefore, the better is the vessel discretization.

The example presented in Fig. 2 represents that generated panel has four points that also belong, at the same time, to other panels. The panel's format depends on its four points disposition. Generally, they do not define a plan, due to hull's irregularities and prominences as well as hull fairing, but as all calculations are made with vectors, the panel's format does not affect the results.

Once the panel's configuration has just been presented, let us define the normal vector. For this, consider now the Fig. 3.

Using as reference the panel in Fig. 3 as well as the coordinate system showed in the same figure, it can be noted that the only way to have a normal vector pointing out of the hull is if the four panel points order stays exactly as showed in Fig. 4.

The reason why the sense of normal vector must point out of the vessel will be better described in vectorial calculus description.

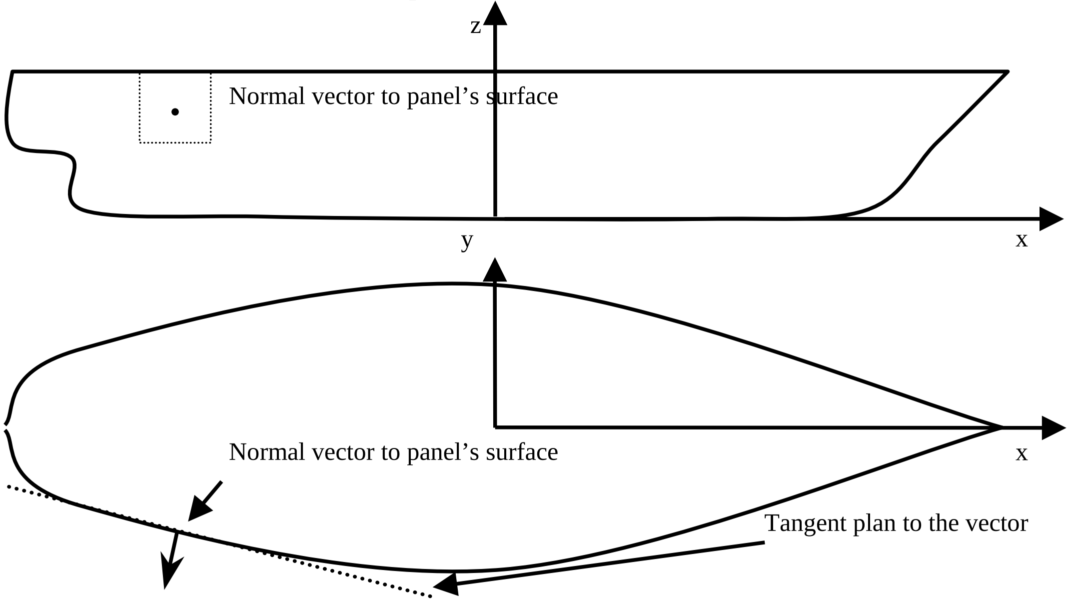


Fig. 3: Panel used in vectorial calculus.

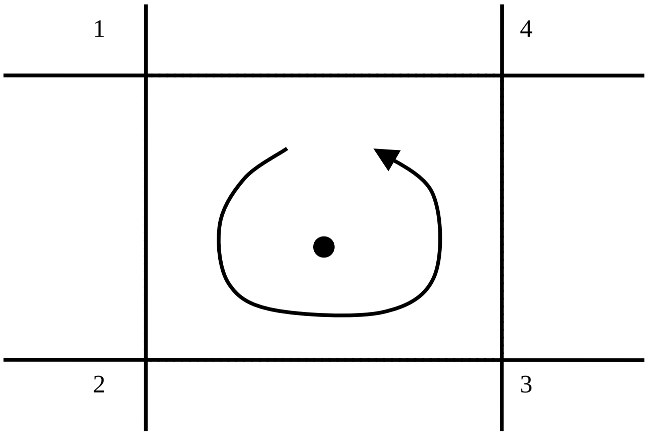


Fig. 4: Points sequence in order to have a vector sense pointing out of the vessel.

Even in relation to panels, NAVSTAB creates them from keel (water line zero) to last deck (last water line) and from stern to bow. Let us verify now how panels creation is made, looking at Fig. 5 which shows a parallelepiped vessel.

Initially, NAVSTAB generates the four points of the panel A in Fig. 5, enumerating in the sequence showed in Fig. 4. Pay attention that it begins at stern and in water line zero.

As vessels are generally considered to be symmetric, another panel, symmetric to panel A in relation to axis x, is created at the other side of the vessel. After finishing points organization in the panel A and its symmetric panel, NAVSTAB starts generating panel B. However, two points of this panel B and that also belong to panel A, they had been created before (when creating panel A). Thus, for panel B, only points number 3 and 4 represented in Fig. 4 are now generated.

For the panels between these first two water lines, the process is exactly the same. In other words, it will be created their points 3 and 4 only. The generated panels' first line ends when NAVSTAB arrives at panel C. For panel D, the correspondent points 2 and 3 had already been created with panel A. Therefore, only points 1 and 4 are now generated. For panel E, the unique missing point is number 4, once all other points have already been created. From this panel on, all other panels will just be constituted by the three existing points and a new one (point 4) created, until the process is over at the last water line on top.

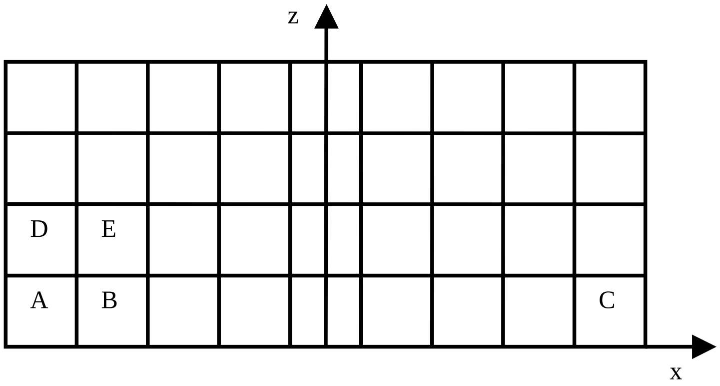


Fig. 5: Panels generation at a hull.

It is easy to realize at this moment that point 4 for panel A is the point 1 for panel B, 3 for D and 2 for E. Hence, the same point belongs to different panels at the same time.

At last, panels at bottom, stern and bow are created when necessary. The vessel is then completely "closed". It is important to remember that NAVSTAB considers a ship as symmetric body and, therefore, identical panels are always generated at the other side (symmetric in relation to axis x in Fig. 3), keeping the normal vector pointing out of the hull. To create the opposite panel, it is required a simple change in the order of the four points, changing point 2 with point 4 in Fig. 4.

As a result, the panel generation and interpolation phase can be summarized in:

- Receive vessel cotes;
- Divide vessel into many stations and water lines, not considering the existing ones;
- *Spline* functions interpolation for each coordinate y of all intermediate stations and water lines;
- Create panels between interpolated stations and water lines, defining a sequence of points with normal vector pointing out of the hull.

2.3 – Final Considerations about Panels Interpolation and Generation

The *spline* function is created with the only purpose of obtaining stations and water lines contours. Both are generated according to user's required precision (number of desired stations and water lines). For example, if user wants to divide a vessel into 20 stations, the program will divide it in 20 stations equally spaced. Their distance depends on vessel's length. The same happens for water lines (depends on depth).

Thus, to generate the required stations and water lines, NAVSTAB calculates *spline* functions which interpolate existing points and delineate the stations and water lines contours.

In relation to vessel's bottom panels, they will be created, when necessary, parallel to plan xy showed in Fig. 3. To better exemplify this case, take a look at Fig. 6. The considered vessel is symmetric and bottom panels are generated, comprehending only the flat part of water line zero (first water line). The same happens with stern and bow, when it is needed to "close" them. The Fig. 7 shows this case.

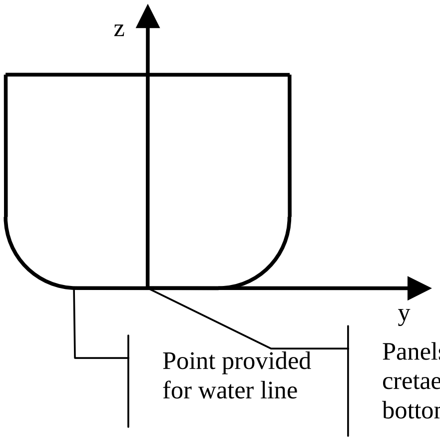


Fig. 6: Panels created at

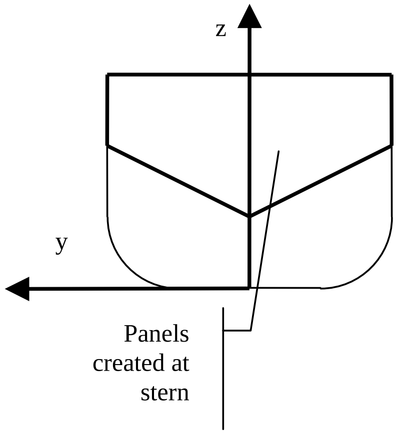


Fig. 7: Panels created at

3 – Obtaining Hydrostatics Curves with Vectorial Calculus

According to Archimedes theorem, it is possible to calculate the actuating forces in an immersed body by using only its geometric description.

However, at this point, there is here a big problem: the calculation based on a three-dimensional geometry can become very complex while the body geometry becomes also more complex. Speaking of floating units geometry, it is almost impossible to describe it analytically because, generally, they are very irregular.

As a consequence, the geometric description of a floating unit into panels, for computational calculation of its main hydrostatic characteristics, becomes a very important tool for a naval project, trying to resolve the almost majority of all existing problems.

3.1 –Theory Explanation

Starting with what was mentioned before, the hydrostatic characteristics of floating units can be calculated based on their immersed geometry. Thus, let us consider a simple example to explain how vectorial calculus can provide those properties.

Consider a cube with a unitary edge whose location is given in relation to a coordinate system xyz, according to Fig. 8.

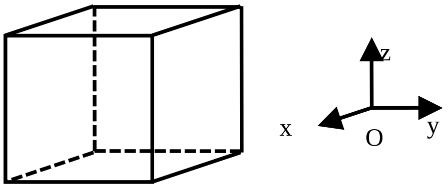


Fig. 8: Cube used for vectorial calculus exemplification.

Let us imagine now that for each cube surface there is a normal vector to it whose modulus is equal to the surface area and pointing out of the cube. Hence, it is easy to realize that the sum of all vectors modulus is equal to the area of cube surface and the vectorial sum of all vectors is equal to zero. From now on, each surface will be considered as a panel.

Now, let us consider that the position of the center of surface for all surfaces is known and the vector that has as modulus the surface area is positioned exactly at this point, as showed in Fig. 9.

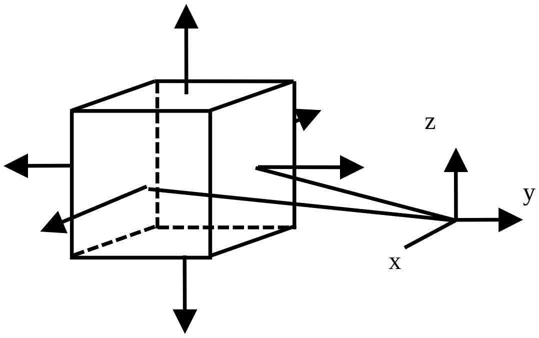


Fig. 9: Cube with the vectors positioned at the center of area for each panel.

Based on linear algebra, it is possible to show that if C_i is the center coordinate for each surface *i* in relation to an origin O and A_i is each vector whose modulus is equal to the surface area of the cube (with *i* = 1,..., 6), the parallelepiped volume will be expressed by:

$$V = \frac{\left(\sum_{i=0}^{6} A_i\left(C_i - O\right)\right)_x + \left(\sum_{i=0}^{6} A_i\left(C_i - O\right)\right)_y + \left(\sum_{i=0}^{6} A_i\left(C_i - O\right)\right)_z}{3} \qquad (7)$$

Actually, the volume can be obtained by each individual part of this expression. In order to minimize numerical imprecision, the volume calculation is taken as an average of the vectorial sum in three directions. Extrapolating the obtained result for a generic geometrical shape (n surfaces or n panels), the solid's volume can be written as:

$$V = \frac{\left(\sum_{i=0}^{n} A_i\left(C_i - O\right)\right)_x + \left(\sum_{i=0}^{n} A_i\left(C_i - O\right)\right)_y + \left(\sum_{i=0}^{n} A_i\left(C_i - O\right)\right)_z}{3} \qquad (8)$$

Considering the same presented principle for a solid's volume calculation, the other hydrostatic properties can be determinate for any body as will be shown later.

The reason why the vector must points out of vessel consists on vector-area signal utilization, after applying for hydrostatic properties formulation, as descript in the next topic.

3.2 – Calculation of Hydrostatic Properties of a Vessel

After creating vessel panels, NAVSTAB positions the coordinate system in water line and in the middle of the ship. To explain the reason for using this reference, let us continue analyzing the same cube of Fig. 8 but now, with the global reference repositioned, as showed in Fig. 10.

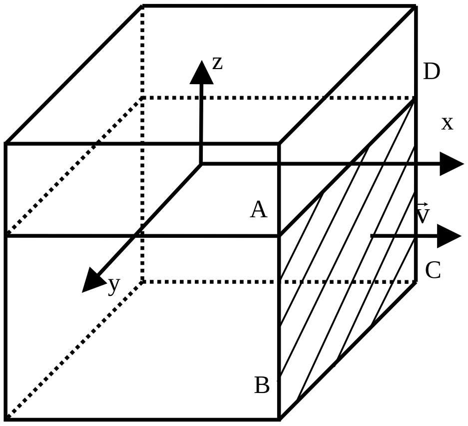


Fig. 10: Cube with the coordinate system repositioned in water line.

As water line is well defined, the program considers for calculation the immersed body only (coordinate z negative). Due to this, all vectors that point out of the body and that have as modulus the panels' area (the vector $\vec{v}$ of Fig. 10 represents one of these vectors), they consider now only the immersed area (panel ABCD), being positioned in the center of the panel (in the example, the same distance to the cube base and to water line) and the area equal to the immersed panel area (panel ABCD).

Using equation (7) for vector $\vec{v}$ in Fig. 10 only, it is noted that the correspondent volume to its face gives one half of total immersed volume. The other half is given by an opposed panel. Thus, considering the five immersed faces, the volume of the body underwater is calculated three times. This can justify equation (7).

The program does not create panels at water line with a vector pointing strictly to axis z positive sense. The Fig. 10 illustrates why it does not exists. The panel's vector at cube bottom can calculate the whole immersed volume.

This simple example with the cube can be extrapolated to a different kind of vessel. The theory used is exactly the same: coordinate system adoption at water line and in the middle of the ship, panels are generated according to required precision, normal vectors to each face are created and equation (8) is applied.

Let us analyze now the panel in Fig. 2. Before starting the vessel's properties calculation, other properties are calculated. Imagine the coordinates for the four points (corners) of the panel in Fig. 2 (p_1, p_2, p_3 e p_4, respectively) in the same sequence showed in Fig. 4.

$$\begin{gathered} p_1 = (x_1,y_1,z_1);\; p_2 = (x_2,y_2,z_2);\; p_3 = (x_3,y_3,z_3);\; p_4 = (x_4,y_4,z_4); \\ \vec{v}_1 = p_2 - p_1;\quad \vec{v}_2 = p_4 - p_1;\quad \vec{v}_3 = p_4 - p_3;\quad \vec{v}_4 = p_2 - p_3. \end{gathered} \tag{9}$$

With these points, it is possible to determinate the projection of areas in three different directions xyz and also the panel's area by the following expressions:

$$\vec{A}^{(x,y,z)} = \frac{\left(\vec{v}_1\times\vec{v}_2\right)+\left(\vec{v}_3\times\vec{v}_4\right)}{2} \tag{10}$$

$$\left|A^{(x,y,z)}\right| = \sqrt{x^2+y^2+z^2} \tag{11}$$

With the vectorial calculus, the distance (C_i – O) between the center of the panel and the coordinate system origin can also be determinate, defining panel properties to be used.

Using equation (11), the wet surface can be calculated by adding the modulus of panels' areas. In other words:

$$S_W = \sum_{i=0}^{n}\left|\vec{A}_i^{(x,\,y,\,z)}\right| \tag{12}$$

Adding the projections of all panels' areas in xy plan, the water line plan area can be calculated as:

$$A_{WL} = \sum_{i=0}^{n} -\vec{A}_i^{z} \tag{13}$$

This formula is also applied for ships with bow or stern bulbs. In order to calculate LCF, it is only necessary to calculate an axis at water plan where the areas ahead and behind it are exactly the same. The following expression shows how to obtain it:

$$LCF = \frac{\sum_{i=1}^{n}\left(-\vec{A}_i^{z} * C^{x}\right)}{\sum_{i=1}^{n} -\vec{A}_i^{z}} \tag{14}$$

The TCF can be obtained with a similar expression used to calculate LCF, only changing the axis used as a reference for the calculus:

$$TCF = \frac{\sum_{i=1}^{n}\left(-\vec{A}_i^{z} * C^{y}\right)}{\sum_{i=1}^{n} -\vec{A}_i^{z}} \tag{15}$$

For almost all vessels, TCF value shall be zero, once vessels are generally symmetric in relation to axis x. With these two properties, it is possible to determinate the longitudinal (I_L) and transversal (I_T) inertial moment. With vectorial calculus the own inertial moment of a panel can be calculated (I_i^L e I_i^T) and we can define the following expressions:

$$I_L = \sum_{i=1}^{n}\left(-\vec{A}_i^{z} * \left(\vec{C}_i^{x} - TCF\right)^2 + I_i^{L}\right) \tag{16}$$

$$I_T = \sum_{i=1}^{n}\left(-\vec{A}_i^{z} * \left(\vec{C}_i^{y} - LCF\right)^2 + I_i^{T}\right) \tag{17}$$

Another important property is the center of buoyancy. As it is situated at the underwater geometrical center of the floating unit, it can be easily obtained by the expression (in relation to the three axes):

$$\mathrm{LCB} = C_B^x = \frac{\sum_{i=1}^{n}\left[\left(\vec{A}_i^x * \vec{C}_i^x\right) * \frac{\vec{C}_i^x}{2}\right]}{\mathrm{Volume}} \tag{18}$$

$$\mathrm{TCB} = C_B^y = \frac{\sum_{i=1}^{n}\left[\left(\vec{A}_i^y * \vec{C}_i^y\right) * \frac{\vec{C}_i^y}{2}\right]}{\mathrm{Volume}} \tag{19}$$

$$\mathrm{KB} = C_B^z = \frac{\sum_{i=1}^{n}\left[\left(\vec{A}_i^z * \vec{C}_i^z\right) * \frac{\vec{C}_i^z}{2}\right]}{\mathrm{Volume}} \tag{20}$$

At last, we can obtain, based on calculated properties, the longitudinal and transversal BM with the expressions:

$$\mathrm{BM}_L = \frac{I_L}{V} \tag{21}$$

$$\mathrm{BM}_T = \frac{I_T}{V} \tag{22}$$

With all these expressions and for different drafts, the hydrostatic curves of a vessel can be constructed.

3.3 – Obtaining Immersed Panels

In this topic it will be explained how immersed panels are searched. Basically NAVSTAB searches for panels that are completely under a considered water line. However, that could exist some of them which are not entirely under water line. For these special panels, a different process is adopted, once only immersed parts of these panels are considered for calculation.

Adopting again the panel in Fig. 4, we can easily identify twelve different cases where the water line can cross a panel. To better visualize all situations, take a look at Fig. 11.

Verify that all possibilities are represented in this figure (water line is represented by a dashed line). We can only have a panel with the following immersed vertices:

1. Points (1), (2), (3) or (4) only;
2. Points (2, 3 and 4); (1, 3 and 4); (1, 2 and 4); (1, 2 and 3);
3. Points (2 and 3); (1 and 4); (1 and 2); (3 and 4).

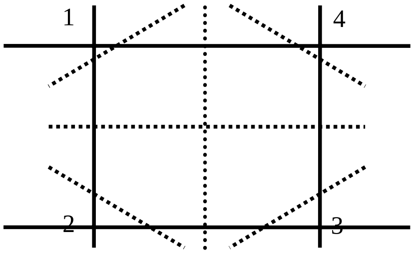


Fig. 11: Possible divisions of a panel by water line.

Thus, NAVSTAB verifies which vertices are immersed and from that moment on, it will consider only the part of the panel that is under water line, creating a new sequence of vertices that will be used to calculate panel's properties.

The panels can also be generated for each draft analyzed by NAVSTAB. Therefore, there are always immersed and emerged panels. An inconvenient would be creating them for each draft to be analyzed, taking more processing time.

3.4 – Implementation of Movements in Relation to xyz Axes

The objective of these movements implementation analysis is evaluating hydrostatic characteristics for floating units.

Although validation for these cases is not presented in this paper, this option is already available in NAVSTAB, representing a vessel possibility to move and rotate in relation to axes x, y and z, as follows:

- *surge* (s): translational movement in relation to x axis;
- *sway* (w): translational movement in relation to y axis;
- *heave* (h): translational movement in relation to z axis;
- *roll* (r): rotational movement in relation to x axis;
- *pitch* (p): rotational movement in relation to y axis;
- *yaw* (y): rotational movement in relation to z axis.

If all panels and their respective normal vectors are positioned in relation to a coordinate system and with a position vector $\vec{X}$ (each coordinate of panel's nodes), the rotation in relation to axes xyz (movements of roll, pitch and yaw, respectively) can be calculated if a matrix M to change system's reference is implemented. This matrix is given by:

$$M = \begin{bmatrix} \cos(p)\cos(y) & \cos(y)\sin(r)\sin(p) - \cos(r)\sin(y) & \sin(y)\sin(r) + \cos(y)\sin(p)\sin(r) \\ \sin(y)\cos(p) & \cos(y)\cos(r) + \sin(y)\sin(p)\sin(r) & \cos(r)\sin(p)\sin(y) - \cos(y)\sin(r) \\ -\sin(p) & \cos(p)\sin(r) & \cos(p)\cos(r) \end{bmatrix} \quad (23)$$

Thus, new panels points coordinate $\vec{X}'$ can be obtained by expression:

$$\left\{\vec{X}'\right\} = \left[M\right].\left\{\vec{X}\right\} \quad (24)$$

The changes in sway and surge movements have only a modification in points' coordinates, not affecting hydrostatic properties. The heave movement changes only draft position, modifying coordinate z. So, these three movements are not considered in matrix M. They are just added or subtracted from points' coordinates (translational movements).

4 – Results Comparison

NAVSTAB validation with results comparison allows a verification of the program accuracy. Therefore, there were made four different tests, with four different vessels and for different load situations (different drafts).

The first comparison and also the simplest one is with a barge, which all properties can be obtained manually and compared with the program's results. Next two comparisons are with tankers whose cotes and hydrostatic curves were obtained in [5]. At last, it was made a test with a platform, comparing program's result with an analytical one.

This last test is actually a program adaptation to a different kind of floating unit. Thus, it was necessary to modify the program's code to adapt it to the platform data given. In this case, the interpolation is not used and the panels were previously specified.

4.1 – Barge comparison

The barge used in this comparison has the dimensions showed in Fig. 12.

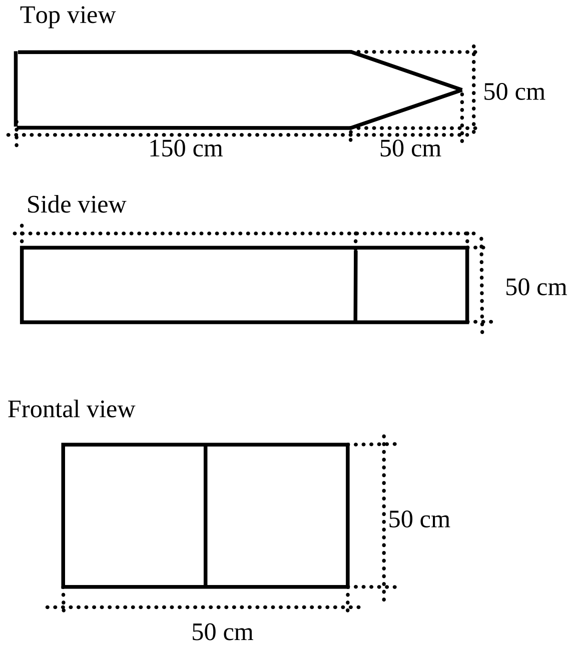


Fig. 12: Drawing of the barge used for first comparison.

Barge main dimensions are showed in Table 1.

Table 1: Barge main dimensions.

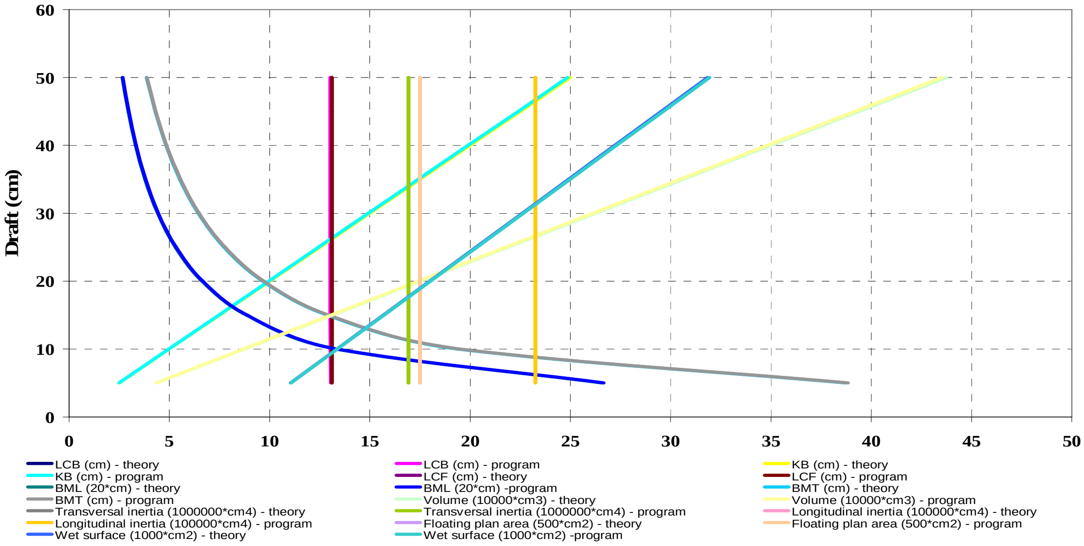
Length	200 cm
Beam	50 cm
Depth	50 cm
Draft	30 cm

The barge cotes were specified to NAVSTAB after taking it in its drawing. As it is a simple vessel, it is not difficult to extract vessel's cotes. The results are showed in Table 2 and the graphic 1 shows hydrostatic curves obtained by NAVSTAB.

Theoretical values and those obtained after NAVSTAB calculation are very close, having as maximum deviation less than 1%. The vessel simple geometry permits that both results (theoretical and achieved) be quite similar.

Table 2: NAVSTAB results for a barge simulation in relation to theoretical ones.

	LCB (cm)			KB (cm)			LCF (cm)		
Draft	Theory	NAVSTAB	**Deviation**	Theory	NAVSTAB	**Deviation**	Theory	NAVSTAB	**Deviation**
5 cm	-11,90	-11,95	**0,42%**	2,50	2,49	**0,40%**	-11,90	-11,90	**0,00%**
10 cm	-11,90	-11,96	**0,50%**	5,00	4,98	**0,40%**	-11,90	-11,90	**0,00%**
15 cm	-11,90	-11,97	**0,58%**	7,50	7,46	**0,54%**	-11,90	-11,90	**0,00%**
20 cm	-11,90	-11,97	**0,58%**	10,00	9,95	**0,50%**	-11,90	-11,90	**0,00%**
25 cm	-11,90	-11,97	**0,58%**	12,50	12,43	**0,56%**	-11,90	-11,90	**0,00%**
30 cm	-11,90	-11,97	**0,58%**	15,00	14,92	**0,54%**	-11,90	-11,90	**0,00%**
35 cm	-11,90	-11,97	**0,58%**	17,50	17,4	**0,57%**	-11,90	-11,90	**0,00%**
40 cm	-11,90	-11,97	**0,58%**	20,00	19,89	**0,55%**	-11,90	-11,90	**0,00%**
45 cm	-11,90	-11,97	**0,58%**	22,50	22,38	**0,54%**	-11,90	-11,90	**0,00%**
50 cm	-11,90	-11,97	**0,58%**	25,00	24,86	**0,56%**	-11,90	-11,90	**0,00%**
	BM_L (cm)			**BM_T (cm)**			**Immersed Volume (cm^3)**		
Draft	Theory	NAVSTAB	**Deviation**	Theory	NAVSTAB	**Deviation**	Theory	NAVSTAB	**Deviation**
5 cm	531,18	533,30	**0,40%**	38,69	38,84	**0,38%**	43750	43576	**0,40%**
10 cm	265,59	266,88	**0,48%**	19,35	19,44	**0,49%**	87500	87077	**0,49%**
15 cm	177,06	177,97	**0,51%**	12,90	12,96	**0,49%**	131250	130578	**0,51%**
20 cm	132,79	133,50	**0,53%**	9,67	9,72	**0,49%**	175000	174079	**0,53%**
25 cm	106,24	106,81	**0,54%**	7,74	7,78	**0,54%**	218750	217581	**0,54%**
30 cm	88,53	89,01	**0,54%**	6,45	6,48	**0,49%**	262500	261082	**0,54%**
35 cm	75,88	76,30	**0,55%**	5,53	5,56	**0,59%**	306250	304583	**0,55%**
40 cm	66,40	66,76	**0,54%**	4,84	4,86	**0,49%**	350000	348084	**0,55%**
45 cm	59,02	59,35	**0,56%**	4,30	4,32	**0,49%**	393750	391586	**0,55%**
50 cm	53,12	53,41	**0,55%**	3,87	3,89	**0,54%**	437500	435112	**0,55%**
	I_L (cm^4)			**I_T (cm^4)**			**Floating surface area (cm^2)**		
Draft	Theory	NAVSTAB	**Deviation**	Theory	NAVSTAB	**Deviation**	Theory	NAVSTAB	**Deviation**
5 cm	23239088	23239096	**0,00%**	1692708	1692708	**0,00%**	8750	8750	**0,00%**
10 cm	23239088	23239096	**0,00%**	1692708	1692708	**0,00%**	8750	8750	**0,00%**
15 cm	23239088	23239096	**0,00%**	1692708	1692708	**0,00%**	8750	8750	**0,00%**
20 cm	23239088	23239096	**0,00%**	1692708	1692708	**0,00%**	8750	8750	**0,00%**
25 cm	23239088	23239096	**0,00%**	1692708	1692708	**0,00%**	8750	8750	**0,00%**
30 cm	23239088	23239096	**0,00%**	1692708	1692708	**0,00%**	8750	8750	**0,00%**
35 cm	23239088	23239096	**0,00%**	1692708	1692708	**0,00%**	8750	8750	**0,00%**
40 cm	23239088	23239096	**0,00%**	1692708	1692708	**0,00%**	8750	8750	**0,00%**
45 cm	23239088	23239096	**0,00%**	1692708	1692708	**0,00%**	8750	8750	**0,00%**
50 cm	23239088	23239096	**0,00%**	1692708	1692708	**0,00%**	8750	8750	**0,00%**
	Wet surface (cm^2)								
Draft	Theory			NAVSTAB			**Deviation**		
5 cm	11059,0			11064,0			**0,05%**		
10 cm	13368,0			13383,1			**0,11%**		
15 cm	15677,0			15702,1			**0,16%**		
20 cm	17986,0			18021,2			**0,20%**		
25 cm	20295,0			20340,2			**0,22%**		
30 cm	22604,1			22659,3			**0,24%**		
35 cm	24913,1			24978,3			**0,26%**		
40 cm	27222,1			27297,4			**0,28%**		
45 cm	29531,1			29616,4			**0,29%**		
50 cm	31840,1			31932,5			**0,29%**		



Graphic 1: Theoretical and calculated hydrostatic curves for a barge.

4.2 – First Tanker Comparison

Main dimensions for the first tanker which was made a comparison with are showed in Table 3.

Table 3: First tanker main dimensions.

Length	180 m
Beam	28 m
Depth	16,8 m
Draft	10,5 m

The tanker cotes were obtained from [5]. The results and deviations are showed in Table 4. The hydrostatic curves are also presented as a graphic (see Graphic 2).

The values obtained by NAVSTAB are also very close to those showed in the reference. However, there is a deviation growth when increasing the draft. The reason for this is the missing information for stern and bow in the reference.

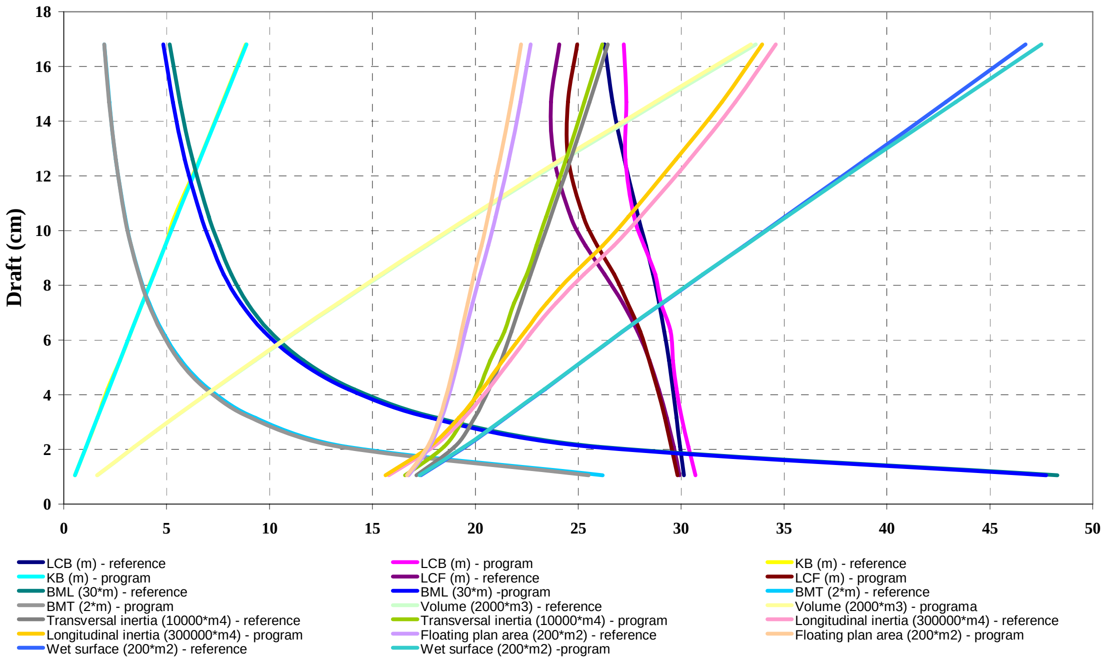
It is also important to say that LCB and LCF deviations are calculated in relation to ship's length.

Table 4: NAVSTAB results for first tanker simulation in relation to reference ones.

	LCB (m)			**KB (m)**			**LCF (m)**		
Draft	Ref. [5]	NAVSTAB	**Deviation**	Ref. [5]	NAVSTAB	**Deviation**	Ref. [5]	NAVSTAB	**Deviation**
1,05 m	5,13	5,70	**0,32%**	0,54	0,54	**0,00%**	4,90	4,82	**0,05%**
2,10 m	4,94	5,35	**0,23%**	1,09	1,09	**0,00%**	4,64	4,53	**0,06%**
3,15 m	4,78	5,03	**0,14%**	1,64	1,64	**0,00%**	4,31	4,21	**0,06%**
4,20 m	4,60	4,78	**0,10%**	2,13	2,18	**2,29%**	3,92	3,85	**0,04%**
5,25 m	4,41	4,61	**0,11%**	2,73	2,73	**0,00%**	3,45	3,45	**0,00%**
6,30 m	4,19	4,50	**0,17%**	3,27	3,28	**0,30%**	2,88	3,01	**0,07%**
7,35 m	3,93	4,03	**0,06%**	3,82	3,83	**0,26%**	2,16	2,41	**0,14%**
8,40 m	3,63	3,75	**0,07%**	4,37	4,38	**0,23%**	1,27	1,75	**0,26%**
9,45 m	3,29	3,20	**0,05%**	4,93	4,93	**0,00%**	0,35	0,92	**0,31%**
10,50 m	2,92	2,72	**0,11%**	5,38	5,49	**2,00%**	-0,35	0,27	**0,35%**
12,60 m	2,24	2,30	**0,03%**	6,6	6,62	**0,30%**	-1,17	-0,49	**0,38%**
14,70 m	1,67	2,34	**0,37%**	7,72	7,75	**0,39%**	-1,32	-0,50	**0,46%**
16,80 m	1,28	2,21	**0,52%**	8,85	8,88	**0,34%**	-0,92	-0,05	**0,49%**
	BM_L (m)			**BM_T (m)**			**Immersed Volume (m^3)**		
Draft	Ref. [5]	NAVSTAB	**Deviation**	Ref. [5]	NAVSTAB	**Deviation**	Ref. [5]	NAVSTAB	**Deviation**
1,05 m	1448,53	1432,34	**1,13%**	52,37	50,99	**2,71%**	3270	3254	**0,51%**
2,10 m	779,84	769,18	**1,39%**	27,46	26,84	**2,31%**	6912	6882	**0,44%**
3,15 m	544,88	535,50	**1,75%**	18,63	18,23	**2,19%**	10711	10665	**0,43%**
4,20 m	423,55	415,09	**2,04%**	14,14	13,85	**2,09%**	14616	14551	**0,44%**
5,25 m	349,71	341,55	**2,39%**	11,42	11,20	**1,96%**	18603	18516	**0,47%**
6,30 m	301,13	292,04	**3,11%**	9,60	9,50	**1,05%**	22664	22549	**0,51%**
7,35 m	267,76	258,02	**3,77%**	8,30	8,20	**1,22%**	26798	26662	**0,51%**
8,40 m	244,56	233,13	**4,90%**	7,32	7,29	**0,41%**	31008	30841	**0,54%**
9,45 m	227,14	215,64	**5,33%**	6,56	6,54	**0,31%**	35300	35108	**0,55%**
10,50 m	211,84	200,50	**5,66%**	5,96	5,94	**0,34%**	39670	39446	**0,57%**
12,60 m	187,67	176,51	**6,32%**	5,06	5,05	**0,20%**	48621	48326	**0,61%**
14,70 m	169,26	159,18	**6,33%**	4,42	4,40	**0,45%**	57826	57456	**0,64%**
16,80 m	154,34	145,31	**6,21%**	3,93	3,92	**0,26%**	67256	66801	**0,68%**
	I_L (m^4)			**I_T (m^4)**			**Floating surface area (m^2)**		
Draft	Ref. [5]	NAVSTAB	**Deviation**	Ref. [5]	NAVSTAB	**Deviation**	Ref. [5]	NAVSTAB	**Deviation**
1,05 m	4737623	4691888	**0,97%**	171284	165931	**3,23%**	3352	3340	**0,35%**
2,10 m	5390587	5332749	**1,08%**	189843	184746	**2,76%**	3555	3545	**0,28%**
3,15 m	5836355	5760329	**1,32%**	199585	194427	**2,65%**	3673	3661	**0,31%**
4,20 m	6190628	6101213	**1,47%**	206687	201611	**2,52%**	3760	3749	**0,30%**
5,25 m	6505929	6400446	**1,65%**	212498	207310	**2,50%**	3833	3822	**0,30%**
6,30 m	6825017	6724526	**1,49%**	217519	214107	**1,59%**	3902	3888	**0,35%**
7,35 m	7175495	7042970	**1,88%**	222298	218731	**1,63%**	3972	3957	**0,38%**
8,40 m	7583530	7425809	**2,12%**	226937	224988	**0,87%**	4048	4030	**0,43%**
9,45 m	8018312	7853745	**2,10%**	231731	229677	**0,89%**	4126	4114	**0,27%**
10,50 m	8403823	8228709	**2,13%**	236587	234441	**0,92%**	4196	4189	**0,17%**
12,60 m	9124958	8920958	**2,29%**	246187	244043	**0,88%**	4325	4321	**0,08%**
14,70 m	9787807	9593395	**2,03%**	255523	252947	**1,02%**	4439	4443	**0,09%**
16,80 m	10380263	10184069	**1,93%**	264354	261646	**1,03%**	4539	4548	**0,21%**

Table 4: NAVSTAB results for first tanker simulation in relation to reference ones (continuation).

	Wet surface (cm^2)		
Draft	Ref. [5]	NAVSTAB	**Deviation**
1,05 m	3471,8	3458,0	**0,40%**
2,10 m	3903,5	3892,3	**0,29%**
3,15 m	4295,9	4286,8	**0,21%**
4,20 m	4675,5	4669,7	**0,12%**
5,25 m	5050,8	5048,5	**0,05%**
6,30 m	5431,3	5426,2	**0,09%**
7,35 m	5819,7	5815,9	**0,07%**
8,40 m	6214,8	6209,5	**0,09%**
9,45 m	6613,3	6619,3	**0,09%**
10,50 m	7007,0	7023,6	**0,24%**
12,60 m	7792,6	7838,5	**0,59%**
14,70 m	8571,6	8668,1	**1,11%**
16,80 m	9346,7	9502,1	**1,64%**



Graphic 2: Theoretical and calculated hydrostatic curves for the first tanker.

4.3 – Second Tanker Comparison

Main dimensions for the second comparison with a tanker are showed in Table 5.

Table 5: Second tanker main dimensions.

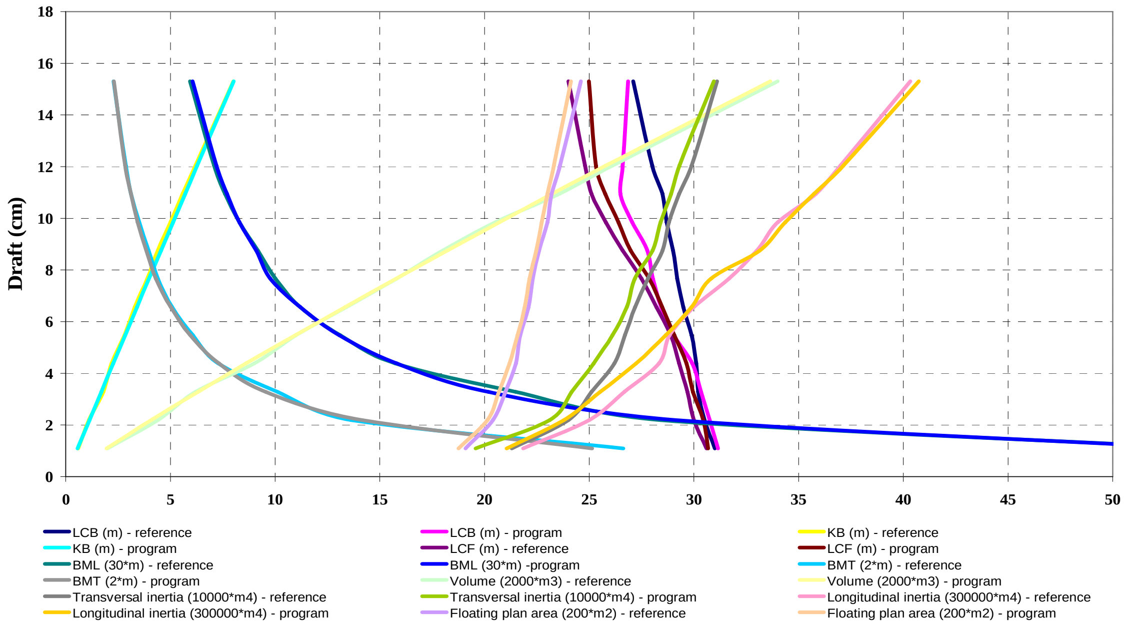
Length	187 m
Beam	29 m
Depth	17,5 m
Draft	10,95 m

The tanker cotes were also obtained from [5]. The results are showed in Table 6. The hydrostatic curves are also presented as a graphic (see Graphic 3).

The same analysis can be made for this second comparison with a tanker. The main difference in this second one case is that the cotes were taken directly from vessel's drawing, giving an additional deviation due to values lecture. However, the results are satisfactory.

Table 6: NAVSTAB results for second tanker simulation in relation to reference ones.

	LCB (m)			**KB (m)**			**LCF (m)**		
Draft	Ref. [5]	NAVSTAB	**Deviation**	Ref. [5]	NAVSTAB	**Deviation**	Ref. [5]	NAVSTAB	**Deviation**
1,09 m	6,00	6,15	**0,08%**	0,60	0,56	**7,14%**	5,60	5,58	**0,01%**
2,19 m	5,50	5,75	**0,14%**	1,10	1,13	**2,65%**	5,00	5,31	**0,17%**
3,28 m	5,20	5,34	**0,08%**	1,80	1,70	**5,88%**	4,70	4,74	**0,02%**
4,38 m	5,10	4,92	**0,10%**	2,20	2,27	**3,08%**	4,30	4,34	**0,02%**
5,47 m	4,90	4,03	**0,48%**	2,80	2,84	**1,41%**	3,90	3,76	**0,08%**
6,56 m	4,50	3,43	**0,59%**	3,30	3,41	**3,23%**	3,20	3,11	**0,05%**
7,66 m	4,20	3,03	**0,65%**	3,90	3,98	**2,01%**	2,50	2,44	**0,03%**
8,75 m	4,00	2,77	**0,68%**	4,40	4,55	**3,30%**	1,60	1,39	**0,12%**
9,84 m	3,70	2,06	**0,91%**	5,00	5,12	**2,34%**	0,80	0,64	**0,09%**
10,94 m	3,50	1,49	**1,12%**	5,59	5,70	**2,02%**	0,10	-0,07	**0,09%**
12,03 m	3,00	1,60	**0,78%**	6,20	6,28	**1,27%**	-0,20	-0,72	**0,29%**
13,10 m	2,70	1,70	**0,56%**	6,90	6,86	**0,58%**	-0,80	-1,14	**0,19%**
15,31 m	2,10	1,85	**0,14%**	8,00	8,02	**0,25%**	-1,00	-1,30	**0,17%**
	BM_L (m)			**BM_T (m)**			**Immersed Volume (m^3)**		
Draft	Ref. [5]	NAVSTAB	**Deviation**	Ref. [5]	NAVSTAB	**Deviation**	Ref. [5]	NAVSTAB	**Deviation**
1,09 m	1637,50	1623,01	**0,89%**	54,00	50,29	**5,89%**	4000	3893	**2,76%**
2,19 m	852,27	874,22	**2,51%**	27,27	28,31	**3,66%**	8800	8163	**7,81%**
3,28 m	645,16	605,85	**6,49%**	20,24	19,12	**5,87%**	12400	12634	**1,85%**
4,38 m	472,22	474,35	**0,45%**	14,56	14,64	**0,58%**	18000	17224	**4,51%**
5,47 m	393,18	392,11	**0,27%**	12,13	11,90	**1,99%**	22000	21924	**0,35%**
6,56 m	335,82	335,84	**0,01%**	10,15	10,03	**1,19%**	26800	26699	**0,38%**
7,66 m	300,63	293,12	**2,56%**	8,80	8,62	**2,06%**	31600	31540	**0,19%**
8,75 m	275,00	272,96	**0,75%**	7,92	7,69	**2,95%**	36000	36435	**1,19%**
9,84 m	248,78	248,91	**0,05%**	7,02	6,86	**2,40%**	41000	41415	**1,00%**
10,94 m	228,06	230,79	**1,18%**	6,22	6,22	**0,07%**	47137	46454	**1,47%**
12,03 m	212,64	216,01	**1,56%**	5,73	5,68	**0,84%**	52200	51573	**1,21%**
13,10 m	199,65	181,56	**1,99%**	5,24	4,60	**0,58%**	57600	56753	**1,49%**
15,31 m	177,94	1623,01	**0,89%**	4,57	50,29	**5,89%**	68000	67299	**1,04%**
	I_L (m^4)			**I_T (m^4)**			**Floating surface area (m^2)**		
Draft	Ref. [5]	NAVSTAB	**Deviation**	Ref. [5]	NAVSTAB	**Deviation**	Ref. [5]	NAVSTAB	**Deviation**
1,09 m	6550000	6317771	**3,68%**	216000	195761	**8,81%**	3820	3755	**1,74%**
2,19 m	7500000	7135966	**5,10%**	240000	231124	**3,84%**	4080	4033	**1,17%**
3,28 m	8000000	7654384	**4,52%**	251000	241598	**3,89%**	4200	4150	**1,21%**
4,38 m	8500000	8170170	**4,04%**	262000	252115	**3,92%**	4300	4256	**1,02%**
5,47 m	8650000	8596681	**0,62%**	267000	260978	**2,31%**	4340	4335	**0,12%**
6,56 m	9000000	8966788	**0,37%**	272000	267888	**1,53%**	4420	4408	**0,27%**
7,66 m	9500000	9244907	**2,76%**	278000	271768	**2,29%**	4460	4459	**0,03%**
8,75 m	9900000	9945041	**0,45%**	285000	280207	**1,71%**	4520	4534	**0,30%**
9,84 m	10200000	10308734	**1,05%**	288000	284117	**1,37%**	4600	4595	**0,12%**
10,94 m	10750000	10720935	**0,27%**	293000	289046	**1,37%**	4640	4665	**0,53%**
12,03 m	11100000	11140295	**0,36%**	299000	293035	**2,04%**	4720	4729	**0,18%**
13,10 m	11500000	12218560	**0,97%**	302000	309534	**0,47%**	4800	4790	**0,21%**
15,31 m	12100000	6317771	**3,68%**	311000	195761	**8,81%**	4920	4909	**0,22%**



Graphic 3: Theoretical and calculated hydrostatic curves for the second tanker.

4.4 – Platform Comparison

The platform used for comparison is a semi-submersible proposed by ITTC for studies of wave forces, presented in [4]. It is composed by eight circular columns, two submersed pontoons and eight structural supports linking columns and other eight linking supports to open deck. The pontoons are rectangular and have their corners rounded. It has as characteristics: project draft of 20 meters and center of gravity position (KG) equal to 17,5 m. Both in relation to pontoon's base (keel). The Fig. 13 shows the platform discretization.

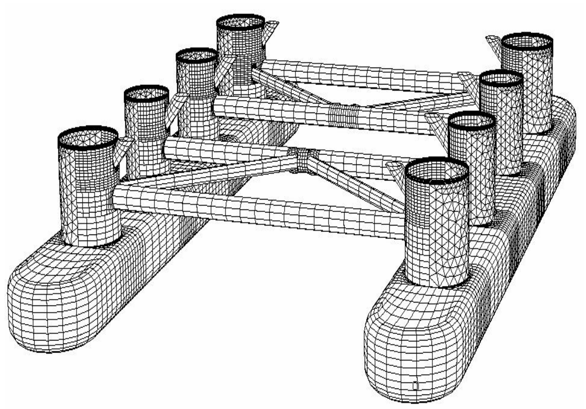


Fig. 13: Platform used for comparison.

The results for project draft and the theoretical ones are presented in Table 7. The coordinate system was positioned in the keel and in the longitudinal and transversal position center of the platform. Although reference system is in the water line and in the middle of the body, it was made a system translation for being possible to compare analytical results and the obtained ones.

Table 7: NAVSTAB results for platform simulation in relation to theoretical ones.

Characteristics	Theory	NAVSTAB	Deviation
KB (m)	6,34	6,29	**0,86 %**
LCB (m)	0,00	0,00	**0,00 %**
LCF (m)	0,00	0,00	**0,00 %**
Immersed Volume (m^3)	34028	33677	**1,04 %**
BM_T (m)	13,65	13,54	**0,81 %**
BM_L (m)	14,26	14,24	**0,14 %**
Floating surface area (m^2)	550,76	542,68	**1,49 %**
Wet surface (m^2)	14913,62	14577,79	**2,30 %**

As the comparison made was only for project draft, the hydrostatic curves were not drawn. Actually, this test was made to show how flexible NAVSTAB is and how to apply a vectorial calculus in order to determinate the platform properties. The analysis method is valid for different kind of floating unit geometry. The results are considered to be satisfactory for all tested cases.

5 – Conclusion

The development of this project helped in a computational tool creation that is able to aid on floating units' project, from a barge one to a complex vessel or even a platform. This work was developed in two distinguished parts. The first part was the interpolation method analysis and the second step was the vectorial calculus.

Dividing the project into different phases it permits a detailed analysis for each individual problem, once they have no directly relationship to be developed. Thus, it was possible to compile the two different parts in only one program, becoming possible to calculate the hydrostatic properties based on a vessel's cotes.

Still in this work, it is possible to add some other calculus, besides a code implementation. It can give more applicability to the tool. According to vectorial calculus theory, it can be applied in many different cases.

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